

# Sukkur IBA **Journal** of Computing and Mathematical Sciences

E-ISSN: 2522-3003

P-ISSN: 2520-0755

Volume: 3 | No: 2 | Jul - Dec | 2019

**Sukkur IBA Journal of Computing and Mathematical Sciences (SJCMS)** is the bi-annual research journal published by **Sukkur IBA University**, Sukkur Pakistan. **SJCMS** is dedicated to serve as a key resource to provide practical information for the researchers associated with computing and mathematical sciences at global scale.

**Copyright:** All rights reserved. No part of this publication may be produced, translated or stored in a retrieval system or transmitted in any form or by any means, electronic, mechanical, photocopying and/or otherwise the prior permission of publication authorities.

**Disclaimer:** The opinions expressed in **Sukkur IBA Journal of Computing and Mathematical Sciences (SJCMS)** are those of the authors and contributors, and do not necessarily reflect those of the journal management, advisory board, the editorial board, Sukkur IBA University press or the organization to which the authors are affiliated. Papers published in **SJCMS** are processed through double blind peer-review by subject specialists and language experts. Neither the **Sukkur IBA University** nor the editors of **SJCMS** can be held responsible for errors or any consequences arising from the use of information contained in this journal, instead errors should be reported directly to corresponding authors of articles.

### ***Mission Statement***

The mission of **Sukkur IBA Journal of Computing and Mathematical Sciences (SJCMS)** is to provide a premier interdisciplinary platform to researchers, scientists and practitioners from the field of computing and mathematical sciences for dissemination of their findings and to contribute in the knowledge domain.

### ***Aims & Objectives***

**Sukkur IBA Journal of Computing and Mathematical Sciences** aims to publish cutting edge research in the field of computing and mathematical sciences.

The objectives of **SJCMS** are:

1. to provide a platform for researchers for dissemination of new knowledge.
2. to connect researchers at global scale.
3. to fill the gap between academician and industrial research community.

### ***Research Themes***

The research focused on but not limited to following core thematic areas:

#### **Computing:**

- Software Engineering
- Formal Methods
- Human Computer Interaction
- Information Privacy and Security
- Computer Networks
- High Speed Networks
- Data Communication
- Mobile Computing
- Wireless Multimedia Systems
- Social Networks
- Data Science
- Big data Analysis
- Contextual Social Network Analysis and Mining
- Crowdsourcing Management
- Ubiquitous Computing

- Distributed Computing
- Cloud Computing
- Intelligent devices
- Security, Privacy and Trust in Computing and Communication
- Wearable Computing Technologies
- Soft Computing
- Genetic Algorithms
- Robotics
- Evolutionary Computing
- Machine Learning

#### **Mathematics:**

- Applied Mathematical Analysis
- Mathematical Finance
- Applied Algebra
- Stochastic Processes

### ***Patron's Message***

Sukkur IBA University has been imparting education with its core values merit, quality, and excellence since its inception. Sukkur IBA University has achieved numerous milestones in a very short span of time that hardly any other institution has achieved in the history of Pakistan. The university is continuously being ranked as one of the best university in Pakistan by Higher Education Commission (HEC). The distinct service of Sukkur IBA University is to serve the rural areas of Sindh and also underprivileged areas of other provinces of Pakistan. Sukkur IBA University is committed to serve targeted youth of Pakistan who is suffering from poverty and deprived of equal opportunity to seek quality education. Sukkur IBA University is successfully undertaking its mission and objectives that lead Pakistan towards socio-economic prosperity.

In continuation of endeavors to touch new horizons in the field of computing and mathematical sciences, Sukkur IBA University publishes an international referred journal. Sukkur IBA University believes that research is an integral part of modern learnings and development. Sukkur IBA Journal of Computing and Mathematical Sciences (SJCMS) is the modest effort to contribute and promote the research environment within the university and Pakistan as a whole. SJCMS is a peer-reviewed and multidisciplinary research journal to publish findings and results of the latest and innovative research in the fields, but not limited to Computing and Mathematical Sciences. Following the tradition of Sukkur IBA University, SJCMS is also aimed at achieving international recognition and high impact research publication in the near future.

**Prof. Nisar Ahmed Siddiqui**

*(Sitara-e-Imtiaz)*

Vice Chancellor, Sukkur IBA University

Patron SJCMS

## *Editorial*

Dear Readers,

It is a pleasure to present to you the Sixth issue of (volume 3, issue 2) of Sukkur IBA Journal of Computing and Mathematical Sciences (SJCMS).

The stunning advances in various fields of science and technology have a profound impact on our lives in almost every sphere of our activity, such as health, agriculture, communication, transportation, and defense. These advances have been driven by an ever-growing volume of exciting discoveries, largely emanating from research community. In order to highlight the future technology challenges, the SJCMS aims to publish cutting-edge research in the field of computing and mathematical sciences for dissemination to the largest stakeholders. SJCMS has achieved milestones in very short span of time and is indexed in renowned databases such as DOAJ, Google Scholar, DRJI, BASE, ROAD, CrossRef and many others. SJCMS is now HEC recognized in Z-Category.

This issue contains the double-blind peer-reviewed articles that address the key research problems in the specified domain. The SJCMS adopts all standards that are a prerequisite for publishing high-quality research work. The Editorial Board and the Reviewers Board of the Journal is comprised of renowned researchers from technologically advanced countries. The Journal has adopted the Open Access Policy without charging any publication fees that will certainly increase the readership by providing free access to a wider audience.

On behalf of the SJCMS, I welcome the submissions for upcoming issue (Volume-4, Issue-1, January-June 2020) and looking forward to receiving your valuable feedback.

Sincerely,  
**Ahmad Waqas, PhD**  
Chief Editor

*Patron Prof. Nisar Ahmed Siddiqui*

*Chief Editor Dr. Ahmad Waqas*

*Editor Dr. Abdul Rehman Gilal*

*Associate Editors Dr. M. Abdul Rehman, Dr. Javed Hussain Brohi*

*Managing Editors Prof. Dr. Pervez Memon, Dr. Sher Muhammad Daudpota*

**Editorial Board**

**Prof. Dr. Abdul Majeed Siddiqui**  
Pennsylvania State University, USA

**Prof. Dr. Gul Agha**  
University of Illinois, USA

**Prof. Dr. Muhammad Ridza Wahiddin**  
International Islamic University, Malaysia

**Prof. Dr. Tahar Kechadi**  
University College Dublin, Ireland

**Prof. Dr. Paolo Bottoni**  
Sapienza - University of Rome, Italy

**Prof. Dr. Md. Anwar Hossain**  
University of Dhaka, Bangladesh

**Dr. Umer Altaf**  
KAUST, Kingdom of Saudi Arabia

**Prof. Dr. Farid Nait Abdesalam**  
Paris Descartes University Paris, France

**Prof. Dr. Asadullah Shah**  
International Islamic University, Malaysia

**Prof. Dr. Adnan Nadeem**  
Islamia University Madina, KSA

**Dr. Jafreezal Jaafar**  
Universiti Teknologi PETRONAS

**Dr. Zulkefli Muhammad Yusof**  
International Islamic University, Malaysia

**Dr. Hafiz Abid Mahmood**  
AMA International University, Bahrain

**Prof. Dr. Luiz Fernando Capretz**  
Western University Canada

**Prof. Dr. Samir Iqbal**  
University of Texas Rio Grande Valley, USA

**Prof. Dr. S.M Aqil Burney**  
IoBM, Karachi, Pakistan

**Prof. Dr. Zubair Shaikh**  
Muhammad Ali Jinnah University, Pakistan

**Prof. Dr. Mohammad Shabir**  
Quaid-i-Azam University Islamabad, Pakistan

**Dr. Ferhana Ahmad**  
LUMS, Lahore, Pakistan

**Dr. Asghar Qadir**  
Quaid-e-Azam University, Islamabad

**Dr. Nadeem Mahmood**  
University of Karachi, Pakistan

**Engr. Zahid Hussain Khand**  
Sukkur IBA University, Pakistan

**Dr. Qamar Uddin Khand**  
Sukkur IBA University, Pakistan

**Dr. Syed Hyder Ali Muttaqi Shah**  
Sukkur IBA University, Pakistan

**Dr. Muhammad Ajmal Sawand**  
Sukkur IBA University, Pakistan

**Dr. Niaz Hussain Ghumro**  
Sukkur IBA University, Pakistan

**Dr. Zarqa Bano**  
Sukkur IBA University, Pakistan

**Dr. Javed Ahmed Shahani**  
Sukkur IBA University, Pakistan

**Dr. Ghulam Mujtaba Shaikh**  
Sukkur IBA University, Pakistan

**Prof. Dr. Florin POPENTIU VLADICESCU**  
University Politehnica in Bucharest

*Language Editor - Prof. Ghulam Hussain Manganhar*

**Project and Production Management**

Mr. Hassan Abbas, Ms. Suman Najam Shaikh, Ms. Rakhi Batra

---

Publisher: **Sukkur IBA Journal of Computing and Mathematical Sciences (SJCMS)**

**Office of Research Innovation & Commercialization – ORIC**

**Sukkur IBA University** – Airport Road Sukkur-65200, Sindh Pakistan

Tel: (092 71) 5644429 Fax: (092 71) 5804425 Email: [sjcms@iba-suk.edu.pk](mailto:sjcms@iba-suk.edu.pk) URL: [sjcms.iba-suk.edu.pk](http://sjcms.iba-suk.edu.pk)

---

## Contents

Performance Comparison of SVM and its Variants for the Early Prognosis of Breast Cancer <i>Talha Ahmed Khan, Muhammad Alam, Zeeshan Shahid, M.S. Mazliham</i>	(1-8)
Crime Mapping in Gujrat, Pakistan using GIS <i>Syeda Ambreen Zahra</i>	(9-24)
Implementation of Kanban Techniques in Software Development Process: An Empirical Study Based on Benefits and Challenges <i>Muhammad Noman Riaz</i>	(25-36)
A Modified 3-Point Adams Block Method of the Variable Step Size Strategy for Solving Neutral Delay Differential Equations <i>Ubaidullah Yashkun, Nurul Huda Abdul Aziz</i>	(37-45)
Solution of Maintenance Strategy Selection Problem by using Modified Fuzzy TOPSIS for of Material Handling Equipment <i>Alia Kausar, Sana Akram, Muhammad Farhan Tabassum, Adeel Ahmad, Shahzaman Khan</i>	(46-54)
A Review of Virtual Touch Interaction Systems <i>Rizwan Ali Abro, Ahsanullah Abro, Sana Fatima, Sarang Shaikh, Sindhu Abro</i>	(55-62)
Paper Submission & Formatting Guidelines	63

## Performance Comparison of SVM and its Variants for the Early Prognosis of Breast Cancer

Talha Ahmed Khan<sup>1,2</sup>, Muhammad Alam<sup>3,4</sup>, Zeeshan Shahid<sup>5</sup>, M.S. Mazliham<sup>6</sup>

---

### Abstract:

Breast cancer has become a leading cause of women death in this era. Breast cancer is very common in various countries including Pakistan. Early identification of the breast cancer or tumor is the only way for the rapid treatment and cure. An imaging approach named as mammography has performed tremendous job in the field of medical to detect the cancer tumors on early basis with less false alarm rate. Breast cancer has two types of tumors a) Benign and b) Malignant. Malignant is acknowledged as cancer tumor as it spread and grow rapidly inside the tissues. Detection of Malignant tumor is very complex in dense breast as it is covered and linked with the milk glands, ducts and other related tissues. Therefore, machine learning and artificial intelligence approaches were needed as mammographic images required edge detection, image enhancement and image processing. Various Artificial Intelligence based algorithms have been applied to the clinical breast cancer data set for the early detection of breast tumor. In this research work the clinical data has been collected from the UCI machine learning repository for the classification of breast cancer tumor a) Benign and b) Malignant. Support Vector machine with its variants Kernel, Gaussian Kernel and Sigmoid Kernel have been applied to the linearly separable breast cancer data set for comparative analysis. Results proved that all the variants of SVM performed better for the breast cancer classification.

**Keywords:** *Benign, Malignant, Breast cancer, dense breast, fatty breast, Support Vector Machine*

---

### 1. Introduction

Digital images based elasto tomography (DIET) was proposed for the robust evaluation of breast tumor. Vibrated breast image was captured and surface motion was recorded to track the tumor. 3D surface motion was

accurately evaluated using this approach. DIET based approach could detect upto 100 mm. of tumor [1]. Gene expression pattern data sets were used to find out the similarities and differences between breast tumor and peripheral blood monocular cells [2]. An intelligent system was proposed comprised of

---

<sup>1</sup> British Malaysian Institute (BMI), Universiti Kuala Lumpur, Malaysia

<sup>2</sup> Usman Institute of Technology, Karachi, Pakistan

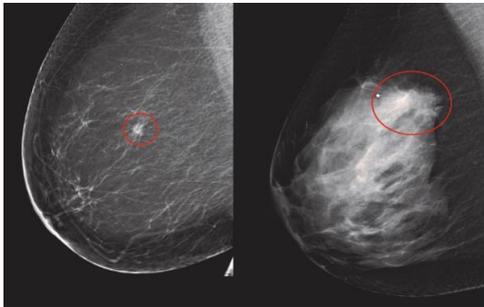
<sup>3</sup> CCIS, Institute of Business Management, Karachi, Pakistan

<sup>4</sup> Malaysian Institute of Information and Technology (MIIT), Universiti Kuala Lumpur Malaysia

<sup>5</sup> Electrical Engineering Department, Institute of Business Management, Karachi, Pakistan

<sup>6</sup> Malaysian France Institute (MFI), Universiti Kuala Lumpur Malaysia

neural network classifier and image processing (ISIBC). The features for the classification have been obtained by applying GLCM algorithm. Standard deviation, mean and entropy were calculated. Filtering, edge detection and morphological techniques were applied to the images for the feature's extraction [3]. Mammographic images were collected from the data bases available on the internet [3]. Image processing method were applied to filter the noise and to enhance the mammographic images for the robust investigation of breast cancer. Adaptive histogram equalization, wavelets and data fusion methods were applied to identify micro-calcifications using LabView [4]. Reproduction of cells is the main indication of the breast cancer. Reproduction can usually be examined by the color of the cells for the antigen Ki-67. Artificial Neural network comprising of the development of multi-layer perceptron network with the combination of feed forward neural network was proposed for the accurate and precise classification of three different cancer genes which are linked with Ki-67. Results proved that unique non-recursive method was very successful to provide the complete detailed genes network with the visualizations [5].



**Fig. 1.** Mammogram image of breast tumor in fatty and dense breast.

Fig. 1 demonstrated the downloaded image of the mammogram which has been downloaded from the mammographic images databases available on the internet. The figure explains that it is very easy to determine the malignant tumor in a fatty breast compared to the dense breast as it is very complex to identify the

malignant tumor in the dense forest. Even a big tumor is very difficult to analyze in a dense breast. Therefore, image processing and filtering techniques are needed to enhance the image quality and features. Mammography is considered as the widely used tool for the detection of the malignant tumor in the breast but in dense breast tissues may overlap each other which causes the confusion for exact investigation of malignant tumor. Tissue overlapping may be minimized by using digital tomosynthesis. Dense breast can be defined as the combination of milk ducts, glands and linked tissues. 4-D virtual prototype breast were designed that were compared to the fourteen imaging approaches [6]. Mammographic filtered images were divided into small grids and features were extracted from these small grids for better classification of Malignant and Benign. Results proved that grid-based pattern evaluation for the doubtful tumor or tissue performed better with the efficiency of 91.67%. The results were also compared with the knowledge-based data base provided by the radiologist [7]. OTSU method was also adopted to reduce the noise and segment the mammogram images to discriminate the changes compared to the normal one [8]. Simplest evolving connectionist system was suggested for the classification of normal, benign and malignant classification. Wisconsin dataset was collected from UCI machine learning repository and sensitivity of 96.02% was achieved [9]. Computer aided diagnostic based approach was developed and 82% sensitivity was achieved [10].

## 2. Problem Statement

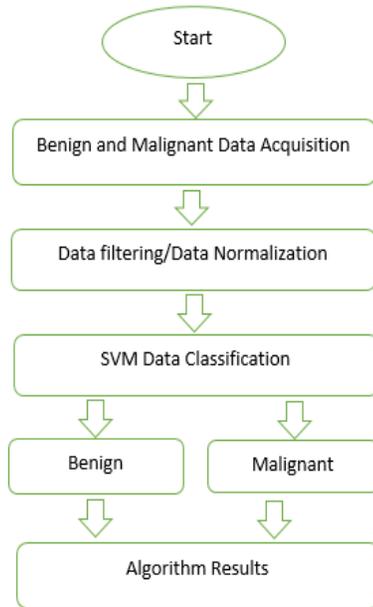
Women normally take the cancer tumor symptoms very lightly due to the the lack of awareness for the malignant tumor which spreads and grow inside the breast tissues. Vigorous classification of malignant and Benign tumor can be acknowledged as very complex due to the similarities between them

which creates confusion [11-14]. Mammography is the most widely used technique for the screening of breast cancer but image filtering, processing and machine learning techniques were needed for the accurate results as identification of malignant tumor in dense breast is very difficult. Skin inflammation and breast cancer have some common symptoms. Breast pain, swelling, and change of skin color are very common symptoms of cancer but people ignore it as they take it as normal skin inflammatory problem [15-17].

spread rapidly. Breast cancer data acquisition was performed and then the data was normalized as the data contained some missed and repetitive values. SVM classifier with the variants of Kernel, Sigmoid and Gaussian Kernel were applied to the breast cancer data set for the classification of Benign and Malignant tumor. The results were compared with each other variants of the SVM. SVM proved to be the competent classifier.

### 3. Methodology

#### 3.1. Fundamental Block Diagram



**Fig. 2.** Fundamental Block diagram

Fig. 1 demonstrated the fundamental block diagram for the breast cancer classification. Breast Cancer tumor can be classified into Benign and Malignant. Malignant is the breast tumor which can grow with the tissues and

#### 3.2. Breast cancer Dat collection

**Table I:** Breast cancer data set [18-19]

Patient ID	Cl. Thick	Cell size U	Cell shape	Adhesion	Cell size	Barc Nuclei	Bland Chromatin	Nucleoli	Mitosis	Class
1002945	5	4	4	5	7	10	3	2	1	2
1015425	3	1	1	1	2	2	3	1	1	2
1016277	6	8	8	1	3	4	3	7	1	2
1017023	4	1	1	3	2	1	3	1	1	2
1017122	8	10	10	8	7	10	9	7	1	4
1018099	1	1	1	1	2	10	3	1	1	2
1018561	2	1	2	1	2	1	3	1	1	2
1033078	2	1	1	1	2	1	1	1	5	2
1033078	4	2	1	1	2	1	2	1	1	2
1035283	1	1	1	1	1	1	3	1	1	2
1036172	2	1	1	1	2	1	2	1	1	2
1041801	5	3	3	3	2	3	4	4	1	4
1043999	1	1	1	1	2	3	3	1	1	2
1044572	8	7	5	10	7	9	5	5	4	4
1047630	7	4	6	4	6	1	4	3	1	4

Table no. 1 displayed all the attributes related to the identification of the breast tumor. Attributes can be explained as:

1. Sample code number: id number
2. Clump Thickness: 1 - 10
3. Uniformity of Cell Size: 1 - 10
4. Uniformity of Cell Shape: 1 - 10
5. Marginal Adhesion: 1 - 10

6. Single Epithelial Cell Size: 1 - 10
7. Bare Nuclei: 1 - 10
8. Bland Chromatin: 1 - 10
9. Normal Nucleoli: 1 - 10
10. Mitoses: 1 - 10
11. Class: (2 for benign, 4 for malignant)

```
breastcancer.head()
```

Patient ID	Cl. th	cell size	Cell shape	Adhesion	Epi. Size	Nuclei	Bl. Chrom.	Nucleoli	Mitoses	Class
0 1002945	5	4	4	5	7	10	3	2	1	2
1 1015425	3	1	1	1	2	2	3	1	1	2
2 1016277	6	8	8	1	3	4	3	7	1	2
3 1017023	4	1	1	3	2	1	3	1	1	2
4 1017122	8	10	10	8	7	10	9	7	1	4

Fig. 3. Imported data set in Python

Data set was imported in python and then normalized to filter the data. Data normalization was performed as data contained some missed and repetitive values.

#### 4. Algorithm Implementation

##### 4.1. Support Vector Machine (SVM)

Support Vector Machine classifier algorithm was applied to the breast cancer data set for the accurate classification of brain tumor. Pandas and Numpy libraries were imported in the Python environment for the implementation of SVM. SVM algorithm creates a boundary by using hyperplanes for the classification. Simple SVM is very useful to classify the linear data. Two hyperplanes were created for this data classification.

H1, H2 are the hyper planes:

$$H1: w \cdot x_i + b = 2 \quad (1)$$

$$H2: w \cdot x_i + b = 4 \quad (2)$$

```
SVC(C=1.0, cache_size=200,
class_weight=None, coef0=0.0,
decision_function_shape='ovr', degree=3,
gamma='auto_deprecated',
kernel='linear', max_iter=-1,
probability=False, random_state=None,
shrinking=True, tol=0.001, verbose=False)
```

	precision	recall	f1-score	support
2	0.83	0.45	0.59	11
4	0.57	0.89	0.70	9
micro avg	0.65	0.65	0.65	20
macro avg	0.70	0.67	0.64	20
weighted avg	0.72	0.65	0.64	20

Fig. 4. SVM Output results

Fig. 4 elaborates that precision of 0.83 and 0.57 have been achieved for the class 2 and class 4 respectively. 0.72 average precision has been achieved by using simple support vector machine.

##### 4.2. Kernel Support Vector Machine (SVM)

Simple Support vector machine is applied to classify the linearly separable data. But for the classification of non-linear data straight line of decision boundary cannot be used. Therefore, a variant of support vector machine named as Kernel was proposed which included the mathematical solution as well for solving the non-linearly separable data. Scikit-learn was needed to implement the Kernel support vector machine with these parameters.

```
SVC(C=1.0, cache_size=200, class_weight=
None, coef0=0.0,
decision_function_shape='ovr', degree=8, ga
mma='auto_deprecated',
kernel='poly', max_iter=-1, probability=False
, random_state=None,
shrinking=True, tol=0.001, verbose=False)
```

	precision	recall	f1-score	support
2	0.00	0.00	0.00	12
4	0.40	1.00	0.57	8
micro avg	0.40	0.40	0.40	20
macro avg	0.20	0.50	0.29	20
weighted avg	0.16	0.40	0.23	20

Fig. 5. Kernel SVM results

Fig. 4 shows that precision, recall, F1-score and support have been calculated to determine the performance of Kernel SVM classifier.

### 4.3. Gaussian Kernel Support Vector Machine (SVM)

$$k(x, x_0) = \exp^{-\frac{1}{2\sigma^2} \|x - x_0\|^2} \quad (3)$$

Gaussian Kernel support vector machine was applied to the breast cancer data set and following results were achieved.

SVC(C=1.0, cache\_size=200, class\_weight=None, coef0=0.0, decision\_function\_shape='ovr', degree=3, gamma='auto\_deprecated', kernel='rbf', max\_iter=-1, probability=False, random\_state=None, shrinking=True, tol=0.001, verbose=False)

	precision	recall	f1-score	support
2	0.60	1.00	0.75	12
4	0.00	0.00	0.00	8
micro avg	0.60	0.60	0.60	20
macro avg	0.30	0.50	0.37	20
weighted avg	0.36	0.60	0.45	20

Fig. 6. Gaussian Kernel SVM results

### 4.4. Sigmoid Kernel Support Vector Machine (SVM)

Sigmoid Kernel can be applied to the data set for the classification by using the following mathematical equation

$$k(x, y) = \tanh(\alpha x^T y + c) \quad (4)$$

SVC(C=1.0, cache\_size=200, class\_weight=None, coef0=0.0, decision\_function\_shape='ovr', degree=3, gamma='auto\_deprecated', kernel='sigmoid', max\_iter=-1, probability=False, random\_state=None, shrinking=True, tol=0.001, verbose=False)

	precision	recall	f1-score	support
2	0.60	1.00	0.75	12
4	0.00	0.00	0.00	8
micro avg	0.60	0.60	0.60	20
macro avg	0.30	0.50	0.37	20
weighted avg	0.36	0.60	0.45	20

Fig. 7. Sigmoid Kernel SVM results

Recall is the measurement of corrected classified values out of the all positive classes. The higher the recall the better the performance. Recall can be measured by using eq. (10).

$$Recall = \frac{Tp}{TP+FN} \quad (5)$$

F-measure can be acknowledged as the comparative analysis or to know the comparison between recall and precision.

$$F - measure = \frac{2 \times Recall \times Precision}{Recall + Precision} \quad (6)$$

The estimation of actual positive out of all positive can be classes can be found be precision.

$$Precision = \frac{TP}{TP+FP} \quad (7)$$

### 4.5. Fine Gaussian Support Vector Machine (SVM)

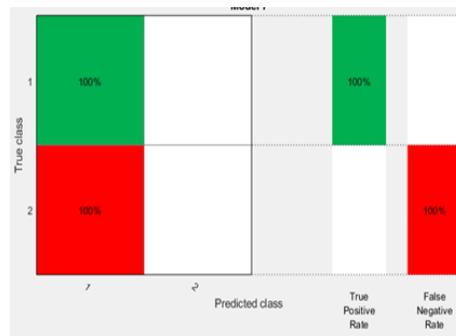


Fig. 8. Confusion Matrix of fine Gaussian SVM

Confusion matrix of fine Gaussian SVM elaborated that the algorithm performed very

poor for the prediction of breast cancer as it classified all classes as class 1. Class 2 was not predicted at all therefore false negative rate was found to be 100% and false positive rate for class 2 was found to be 0%.

### 5. Comparative Analysis

**Table II:** SVM Parametric Evaluation (CLASS 2)

Classifiers	Precision	Recall	F1-Score	Support
SVM	0.83	0.45	0.59	11
Kernel SVM	0.0	0.0	0.0	12
Sigmoid Kernel SVM	0.60	1	0.75	12
Gaussian Sigmoid Kernel	0.60	1	0.75	12

Table II demonstrated that the SVM and its variants' parametric analysis for the classification of class 2. It can be observed that linear support vector machine achieved better results in terms of precision, recall, F1-score.

**Table III:** SVM Parametric Evaluation (CLASS 4)

Classifiers	Precision	Recall	F1-Score	Support
SVM	0.57	0.89	0.70	9
Kernel SVM	0.4	1	0.57	8
Sigmoid Kernel SVM	0	0	0	8
Gaussian Sigmoid Kernel	0	0	0	8

Table III demonstrated that the SVM and its variants' parametric analysis for the classification of class 4. It can be observed that linear support vector machine achieved better results in terms of precision, recall, F1-score.

### 6. Results and discussion

Support vector machine with its variants Kernel, Gaussian Kernel and Sigmoid Kernel have been applied to the data set of breast cancer. All the variants performed better. 0.57 have been achieved for the class 2 and class 4 respectively. 0.72 average precision has been achieved by using simple support vector machine. For the future work SVM can be combined with the Particle swarm optimization in which weights of Support vector machine would be optimized by PSO. The optimization of SVM weights would make the results better and up to the mark for the classification of breast cancer.

### REFERENCES

- [1] T. Botterill, T. Lotz, A. Kashif and J. G. Chase, "Reconstructing 3-D Skin Surface Motion for the DIET Breast Cancer Screening System," in *IEEE Transactions on Medical Imaging*, vol. 33, no. 5, pp. 1109-1118, May 2014. doi: 10.1109/TMI.2014.2304959
- [2] L. He, D. Wang and Z. Guo, "Identification of Potential Non-invasive Biomarkers for Breast Cancer Prognosis and Treatment by Systematic Bioinformatics Analysis," *2015 7th International Conference on Information Technology in Medicine and Education (ITME)*, Huangshan, 2015, pp. 117-120. doi: 10.1109/ITME.2015.28
- [3] A. Helwan and R. H. Abiyev, "ISIBC: An intelligent system for identification of breast cancer," *2015 International Conference on Advances in Biomedical Engineering (ICABME)*, Beirut, 2015, pp. 17-20. doi: 10.1109/ICABME.2015.7323240
- [4] Meharun Nisa S.P and K. Suresh, "Labview implementation of identification of early signs of breast cancer," *2014 International Conference on Electronics, Communication and Computational Engineering (ICECCE)*, Hosur, 2014, pp. 264-267. doi: 10.1109/ICECCE.2014.7086624
- [5] D. Agarwal, M. Kergosien, D. J. Boocock, R. C. Rees and G. R. Ball, "A systems biology approach to identify proliferative biomarkers and pathways in breast cancer," *2014 IEEE International*

- Conference on Bioinformatics and Biomedicine (BIBM)*, Belfast, 2014, pp. 1-7.  
doi: 10.1109/BIBM.2014.6999240
- [6] N. Kiarashi *et al.*, "Development and Application of a Suite of 4-D Virtual Breast Phantoms for Optimization and Evaluation of Breast Imaging Systems," in *IEEE Transactions on Medical Imaging*, vol. 33, no. 7, pp. 1401-1409, July 2014.  
doi: 10.1109/TMI.2014.2312733
- [7] P. Swapnil, E. Pandey, J. R. Yathav, A. Baig and A. Bailur, "Region Marking and Grid Based Textural Analysis for Early Identification of Breast Cancer in Digital Mammography," *2016 IEEE 6th International Conference on Advanced Computing (IACC)*, Bhimavaram, 2016, pp. 426-429.  
doi: 10.1109/IACC.2016.85
- [8] J. Kamalakannan, P. V. Krishna, M. R. Babu and K. D. Mukeshbbhai, "Identification of abnormality from digital mammogram to detect breast cancer," *2015 International Conference on Circuits, Power and Computing Technologies [ICCPCT-2015]*, Nagercoil, 2015, pp. 1-5.  
doi: 10.1109/ICCPCT.2015.7159454
- [9] E. B. Nababan, M. Iqbal and R. F. Rahmat, "Breast cancer identification on digital mammogram using Evolving Connectionist Systems," *2016 International Conference on Informatics and Computing (ICIC)*, Mataram, 2016, pp. 132-136.  
doi: 10.1109/IAC.2016.7905703
- [10] S. Pathan, P. C. Siddalingaswamy, L. Lakshmi and K. G. Prabhu, "Classification of benign and malignant melanocytic lesions: A CAD tool," *2017 International Conference on Advances in Computing, Communications and Informatics (ICACCI)*, Udupi, 2017, pp. 1308-1312.  
doi: 10.1109/ICACCI.2017.8126022
- [11] I. E. Magnin, D. Vray and A. Brémond, "Early detection of breast cancer using computer assisted diagnosis," *1992 14th Annual International Conference of the IEEE Engineering in Medicine and Biology Society*, Paris, 1992, pp. 849-850.
- [12] R. D. Ghongade and D. G. Wakde, "Detection and classification of breast cancer from digital mammograms using RF and RF-ELM algorithm," *2017 1st International Conference on Electronics, Materials Engineering and Nano-Technology (IEMENTech)*, Kolkata, 2017, pp. 1-6.  
doi: 10.1109/IEMENTECH.2017.8076982
- [13] P. Král and L. Lenc, "LBP features for breast cancer detection," *2016 IEEE International Conference on Image Processing (ICIP)*, Phoenix, AZ, 2016, pp. 2643-2647.  
doi: 10.1109/ICIP.2016.7532838
- [14] B. Hela, M. Hela, H. Kamel, B. Sana and M. Najla, "Breast cancer detection: A review on mammograms analysis techniques," *10th International Multi-Conferences on Systems, Signals & Devices 2013 (SSD13)*, Hammamet, 2013, pp. 1-6.  
doi: 10.1109/SSD.2013.6563999
- [15] T. Kao *et al.*, "Regional Admittivity Spectra With Tomosynthesis Images for Breast Cancer Detection: Preliminary Patient Study," in *IEEE Transactions on Medical Imaging*, vol. 27, no. 12, pp. 1762-1768, Dec. 2008.
- [16] D. A. Woten, J. Luth and M. El-Shenawee, "Interpreting Artificial Neural Networks for Microwave Detection of Breast Cancer," in *IEEE Microwave and Wireless Components Letters*, vol. 17, no. 12, pp. 825-827, Dec. 2007.  
doi: 10.1109/LMWC.2007.910466
- [17] P. M. Meaney, M. W. Fanning, D. Li, S. P. Poplack, and K. D. Paulsen, "A clinical prototype for active microwave imaging of the breast," *IEEE Trans. Microw. Theory Tech.*, vol. 48, no. 11, pp. 1841-1853, Nov. 2000.  
doi: 10.1109/MCS.2009.932223
- [18] Wolberg, W.H., & Mangasarian, O.L. (1990). Multisurface method of pattern separation for medical diagnosis applied to breast cytology. In *Proceedings of the National Academy of Sciences*, 87, 9193-9196.
- [19] Zhang, J. (1992). Selecting typical instances in instance-based learning. In *Proceedings of the Ninth International Machine Learning Conference* (pp. 470--479). Aberdeen, Scotland: Morgan
- [20] T. Khan *et al.*, "Foreign objects debris (FOD) identification: A cost effective investigation of FOD with less false alarm rate," *2017 IEEE 4th International Conference on Smart Instrumentation*,

- Measurement and Application (ICSIMA), Putrajaya, 2017, pp. 1-4.
- [21] Talha Ahmed Khan, Kushsairy Kadir, Muhammad Alam, Zeeshan Shahid and M.S. Mazliham: "Geomagnetic Field Measurement at Earth Surface: Flash Flood Forecasting using Tesla Meter", *Proc. of the International Conference on Engineering Technologies and Technopreneurship (IEEE-ICE2T 2017) 18-20 September 2017, Kuala Lumpur, Malaysia*
- [22] T. A. Khan, M. Alam, K. Kadir, Z. Shahid and S. M Mazliham, "A Novel Approach for the Investigation of Flash Floods using Soil Flux and CO<sub>2</sub>: An Implementation of MLP with Less False Alarm Rate," *2018 2nd International Conference on Smart Sensors and Application (ICSSA)*, TBA, Malaysia, 2018, pp. 130-134
- [23] Talha Khan, Muhammad Alam, Kushsairy Kadir, Sheraz Khan, M.S Mazliham, Faraz Shaikh, Syed Faiz Ahmed, Zeeshan Shahid" An Implementation of Electroencephalogram signals acquisition to control manipulator through Brain Computer Interface ", *2<sup>nd</sup> IEEE International Conference on Innovative research and development 2019 (IEEE-ICIRD), 28-30 June 2019, Jakarta, Indonesia*

## Crime Mapping in Gujrat, Pakistan using GIS

Syeda Ambreen Zahra<sup>1</sup>

---

### Abstract:

In previous years, crime has turned into a broad-spectrum term in Pakistan. Street crime in Pakistan is in the high range. This needs a measurable research on crime mapping. Research about crime with its spatial and temporal distribution is important because data about crime Events is one of the most necessary need to Defends against crime [19]. Crime mapping is figure out how crimes are spread evenly over the zone. GIS plays an effective role in mapping of crime. This paper puts on the diverse utilities of GIS to Identify the hot, low crime area and safe area of Gujrat, Pakistan in addition to encourage the advancement of investigation and safety strategy for policing using GIS. This Research using GIS present better investigation for crime mapping in Gujrat, Pakistan so by using GIS in police department can be successfully applied on all police stations for better safety. GIS is technological gadgets which normally improve policing process as it simplifies the cognitive metrics and relationships between spatial localities. Studies reveal that GIS can holistically be integrated with crime investigating systems of police. This indicates that GIS is feasible, adoptable and adaptable tool in crime Deducting systems. This study aims to find out generic crime activity patterns of Gujrat using applications of GIS in crime patrolling and reduction activities. This study also aims to ensure that GIS can improve crime prediction and proactive measures for crime control in Gujrat. It will not only highlight the hot spots of crime.

*Keywords: Crime mapping, Spatial, GIS, Hotspot Spatial Temporal analysis, Crime Forecasts, Hotspot Mapping, Positional Errors, Predictive Ability, Temporal Information*

---

### 1. Introduction

Theories reveal that crime show that the threat of crime raises with the metropolis and societal exchange. Increase in population of towns is directly proportional to crime rate it means crime rate increase with increase in population in an area. New towns are under construction to facilitate growing population, but existing town and localities are under immense pressure of dense population. The density of population of any area is directly connected with the facilities available in that

area. Main or old cities are victims of dense or extra dense population and people due to typical mind set are reluctant to move to new localities. The Extra density of population in cities has created many social and economic problems and crime is one of them. (Gupta, Rajitha, et al, 2012).

GIS applications have been proven extraordinary partner to plot crime styles. It has practically pinpointed the impacts of crime in social, economic and other contexts. Police

---

<sup>1</sup> *Computer Science and Information Technology, University of Lahore, Gujrat, Pakistan*  
Corresponding Author: msituol@gmail.com

services have preferred using and applying GIS applications rather than old mapping pins. GIS is technological gadgets which normally improve policing process as it simplifies the cognitive metrics and relationships between spatial localities. Studies reveal that GIS can holistically be integrated with crime investigating systems of police. This indicates that GIS is feasible, adoptable and adaptable tool in crime Deducting systems. This study aims to find out generic crime activity patterns using applications of GIS in crime patrolling and reduction activities. This study also aims to ensure that GIS can improve crime prediction and proactive measures for crime control. It will not only highlight the hot spots of crime.

Therefore, any crime control system for Pakistan will be more beneficial if it is integrated with GIS crime control applications. GIS brings into use software applications and crime database to fetch, measure and analyze the facts about criminal activities. So GIS applications for Gujrat from geographical and exorcism analogical view. A GIS uses the base map to investigate and observe the mapping of crimes localities by adapting virtual base map. Initially the crime database is populated by getting information from the City Police Liaison Committee (CPLC). To digitalized the excessive decision of satellite statistics that will be employed on overlays of bottom rape. Cutting-Edge GIS technology assists in collecting spatial and temporal scopes of a crime regarding the place of crime. GIS based produced maps are given a deep analysis of crime by removing hassle areas. It hot spots the output of all the map activities and extract a variety of relationships of spatial element. (T. Balogan et al 2016). GIS applications have been proving extraordinary partner to plot crime styles. It has practically pinpointed the impacts of crime in social, economic and other contexts. Police services have preferred using and applying GIS applications rather than old mapping pins. GIS is technological gadgets which normally improve policing process as it simplifies the cognitive metrics and relationships between spatial localities. Studies reveal that GIS can

holistically be integrated with crime investigating systems of police. This indicates that GIS is feasible, adoptable and adaptable tool in crime eradicating systems. This study aims to find out generic crime activity patterns using applications of GIS in crime patrolling and reduction activities. This study also aims to ensure that GIS can improve crime prediction and proactive measures for crime control. It will not only highlight the hot spots of crime, but identification of peak hours of criminal activities. In first section literature review is discussed and define different way of mapping in past. in next section defined proposed methodology for crime mapping next limitation of Research is explained and in last section conclusion and future work is define

## 2. Literature Review

GIS started to become part of crime patrolling in 1970s. Its major aim is to measure the spatial modules in the cartographic paradigm because conventional paper-based mapping was no longer possible due to increasing economic growth, urbanization and crime rate. Popular GIS turned into the Geographic Information System and it was intended to organize statistics in a natural way of Canada (1965). As suggested with the aid of Longley Good child et al (2005), a near modern approach was adopted by the census Bureau by USA to measure the behavior of a population which could be helpful in eradication of criminal activities form society. (M. Adevijk et al 2014)

GIS was implemented in the army and then its applications were brought into a Civilian GIS in the U.S and intelligence imagery software of the Nineteen Sixties (Shamsi, 2005) was started to measure crime ration for different spatial localities. Zero Now GIS has been identified by means of many types of research numerous topics were mentioned by means of the use of this era like spatial, temporal modeling, segregation and classification of facts to manipulate the large data set. MacEachren et al. Introduced the map animation and multivariate demonstration applications for information with the aid of

concentrating the time series by way of the usage of fitness, surroundings, and demographic information. United States environmental guard organization also exercises in GIS surroundings to correlate it with scientific fashions if you want to make or produce 3-d visualized maps. GIS also used a database gadget. They mapped crime rate and its relationship with new immigrants and their impact on the social life of permanent residents and economy.

This research was prolonged to a long duration of five decades. They formulated the analysis of crime mapping for the first half of the nineteenth century. Research conducted in 1979 by The Ecological Technique by Stanley Baldwin, established relationship chart and suggested a detailed distribution of crime map along with special points and localities from where criminals lived and appeared in crime. It was a valuable research by Shaw & McKay which different parameters for analysis and detect crimes. Morris (1957) focused towards the relationship of the vicinity and regular crime regions where criminal's mindsets were used to live during the period of 1950s and 1960s. It has molded the attitude of criminology as major components of criminology had started to be measured and analyzed by the crime patrolling department. In early 1970s, trading and economic parameters were carried forward as major attributes to detect and prevent criminology. As discussed above, the environment and weather conditions were also giving advantage to criminal mind set therefore they were included as variables to analysis criminology and it was named as Environmental criminology. It played an important role in crime prevention. O. Newman (1972) in his book "Defensible Space" reveals that detection of unlawful in any locality could also prove helpful in prevention of crime. It was a proactive approach which could prove a handy approach to detect a crime from an unlawful gathering of getting together.

It was a sociological angle to prevent crime. Another major angle to detect and prevent criminology an area or spatial locality, it was

named as geographical criminology. Environmental criminologist prepared patterns of regular crimes in some areas and movement of people in spatial localities to find out the relationship between crimes and movement of people. Gujrat is considered as the commercial hub of Punjab province and social position and affordability of its citizens is more than reasonable. This is an old city and it has its special history related to The Great Alexander and the River Chenab. It is a city of 16 Lac heads. It has two major problems: crime and care. Criminals are getting the benefit of large population residing in the congested place. There are many localities where not only visitors' even residents think themselves unsafe and insecure in terms of life and valuables. Violence and street crime have become the norm of these areas and writ of law is negligible in absence of security personals. Gujrat is not only economically strong city; it is politically also very active. Political gangs often support different plundering and looting gangs. This city has been measured with about 128% prolonged sectarian and political violence. Kidnapping and rape are also top list crimes which are supported by different groups in the city. (Persistence,2012)

#### **A. Global Distribution of Crime**

Criminological examine becoming a nicer preferred device for a competitive crime. Due to its more capability of GIS can without problems recognize hassle regions, whilst showing them in a manner that makes an easily understandable map using visual means. Geographic Information Science (GIS) makes use of replicated maps and geography as an interface for integrating annual having access to large portions of the area primarily based records. The GIS allows police personnel to plot successfully for an emergency answer, determine mitigation priorities and to take a look at ancient activities, in forecast destiny activities.

While within the various elements of Pakistan there may be a mountaineering trench of crime is delivering into being. Although Low-

crimes societies like Islamabad and Multan are functioned lofty amongst road crime rates, other international country, i.e. Japan was also suffering an upward push in crime. Pakistan had a higher murder fee from 1980-2000, permitting. To. 2000 determine the rate of homicides consistent with 100.000 human beings changed into still superior to all international localities; the murder rate is also very high in South Africa as compared to other countries in the continent. Graph of rape cases is also increasing in European states. But by means of 2000, Canada took the lead in lowest rape cases registered due to implementation of GIS with crime patrolling. Robbery has been on a sound decline in Pakistan over two decades.

According to the 2000 ranking, Portugal, England, Wales and Spain have extra pronounced robberies than US countries. Germany stands at first in low robbery crimes. Italian and French localities along with Middle Eastern states, is most of the most serif us assets crime, at the same time as its far decrease in the United States today than it became in 1980. As of 2000, America had decreased housebreaking charges than Finland, Australia, Denmark, England, Wales and Canada.

**B. Significance of the Study**

Crime control related research regarding Gujrat is highly valuable for all law enforcement and crime control agencies to detect and prevent crime and aid in coping with crimes extra meaningfully and effectively. This could without a doubt assist Gujrat’s citizens to stay in Gujrat with concord and peace. The set pursuits of this study are supportive for the position of police sources appropriate, in addition, to assist out in recognizing the various elements which can be chargeable for the high crime charged in the study region. This GIS and CRIMINOLOGY-BASED research will deliver, the delivery to various strategies and rules for supervisory the crime. GIS programs are used everywhere globally to incorporate crime detection and

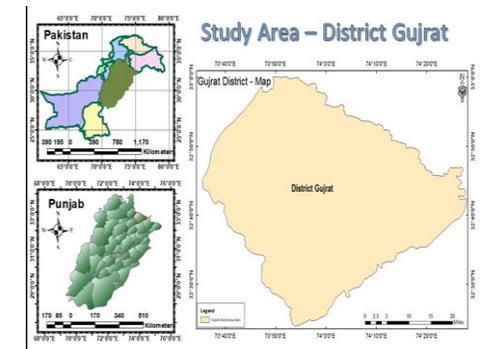
prevention strategies. The scarcity of police resources in terms of human and physical resources is under stress due to frequent upward thrust in crime.

**3. Proposed Methodology**

**A. Study Area**

Gujrat is district of Punjab province in Pakistan. Gujrat is an ancient district located between two famous rivers, the Jhelum and Chenab. It is bounded on the northeast by Mirpur, on the northwest by the River Jhelum which splits it from Jhelum District.

**B. Map of Gujrat**



**Fig 1.** Study Area-Map of Gujrat District

**C. Data Used in Study**

First of all, we got approval letter from university for collecting crime data of police stations to complete our research work. After that we got 23 police station names with personal number for crime data collection. We personally visit every police station with advocate Syed irtza Haider Sharazi as a legal advisor and demand for crime data, but they denied for giving us the data and said crime data is always secret we can't give you in hard form and then we request, kindly mark crime data with us on a google earth map for completion of our research work so that some police station mark data with us on a google earth map and also some police station give

us data in excel sheets, Google Earth ArcGIS. These two tools have different features Google earth is one of the most common applications used for locating places on the earth’s surface. ArcGIS on the other hand, is used to map out locations on the earth’s surface and then carry out various analyses on the selected place on the earth’s surface using a variety of tools available in the program.

**D. Methodology Architecture**

The following diagram shows the methodology steps flow. In the very 1<sup>st</sup> step, we prepared the crime dataset including area boundary, number of police stations, marking crime data from police stations and converting KML to ArcGIS and applying grid on Gujrat district including different police stations. In the 2<sup>nd</sup> step, we performed GIS Analysis by joining spatial crime data with grid and then conversion of grid to raster and the classification of crime into 5 classes. In 3<sup>rd</sup> step, we obtained results in categorical form of safe zone, high crime, middle level crime and low-level crime. And in the last step we presented the the obtained results in the form of crime map. In experimental study section, we explained every single step of proposed methodology.

**4. Experimental Study**

In the data preparation mark the Gujrat district boundary on the Google map after that divided in tehsil and mark police station of Gujrat on the district Gujrat map areas. With the help of police we mark crime points on the map and we covert Google earth file of crime data Gujrat into ArcGIS map format And applied Grid on Gujrat area 1Sqkm by 1SqKm on district Gujrat area for counting Crime ratio in 1 grid In GIS analysis applied to spatially join Crime With Grid for a clear representation of Data And After that Convert Grid into raster because Grid Store Boundary into points, line, polygon and Raster (Vector) store data in an aerial photograph, a satellite image. And Classify Crime into Five classes to showing

high alert area and low crime alert areas and so on.

In Result obtain area shows crime Distribution on safe zone, low crime, medium crime, and High Crime.In final data presentation crime map of Gujrat district is prepared

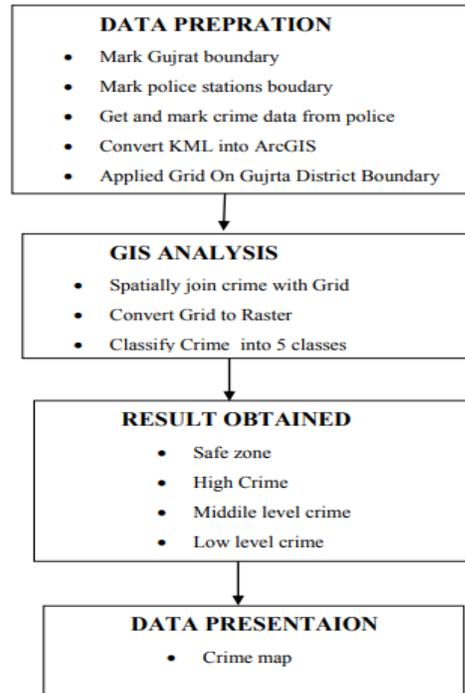


Fig 2. Architecture Diagram

**5. Limitation**

Even though this research final crime map is very useful to the police, but it does not fully cover crimes of Gujarat because of their safety and security issues. Some of the cons that come with crime maps are that sometimes they can have weighted anecdotal information. Some crime mark in police station. Some crimes are not reported. This can sometimes result in crime mapping having a bias on historical problems as opposed to emergent patterns. Therefore, deployment errors can easily occur with the Gujarat police departments.

### 6. Gujrat Police Station

List of Police Stations District Gujrat

Sr.	City Name	Tehsil Name	Police Station Name	Location	
				Latitude	Longitude
1	Gujrat	Gujrat	Daulat Nagar Police Station	32.749272	74.070668
2	Gujrat	Gujrat	Jalapur Jattan Police Station	32.640191	74.208335
3	Gujrat	Gujrat	Sadar Police Station Gujrat	32.569375	74.099828
4	Gujrat	Gujrat	Lorry Ada Police Station	32.565545	74.078068
5	Gujrat	Gujrat	Police Station "A" Division	32.570272	74.074064
6	Gujrat	Gujrat	Khawajgan Police Station	32.576395	74.078756
7	Gujrat	Gujrat	Motorway Police Station	32.541336	74.004236
8	Gujrat	Gujrat	Kunjab Police Station	32.523992	73.976479
9	Gujrat	Gujrat	Mangowal Police Station	32.494986	73.891864
10	Gujrat	Gujrat	Shadiwal Police Station	32.512452	74.035456
11	Gujrat	Gujrat	Civil Line Police Station	32.587171	74.076468
12	Gujrat	Gujrat	Shaheen Police Choki	32.559865	74.046552
13	Gujrat	Gujrat	Karinwala Police Station	32.749238	74.279148
14	Gujrat	Gujrat	Purana Tanda Police Station	32.702193	74.369416
15	Gujrat	Kharian	Kharian Cantt Police Station	32.804591	73.882252
16	Gujrat	Kharian	Kharian Police Station	32.815325	73.861223
17	Gujrat	Kharian	Kakrali Police Station	32.840309	74.063518
18	Gujrat	Kharian	Guliana Police Station	32.811953	73.969853
19	Gujrat	Kharian	Dinga Police Station	32.643775	73.721246
20	Gujrat	Sarai Alamghir	Sarai Alamgir Police Station	32.898516	73.757683
21	Gujrat	Sarai Alamghir	Bulani Police Station	32.92486	73.931981
22	Gujrat	Sarai Alamghir	Goteriiala Police Station	32.931069	74.01683
23					

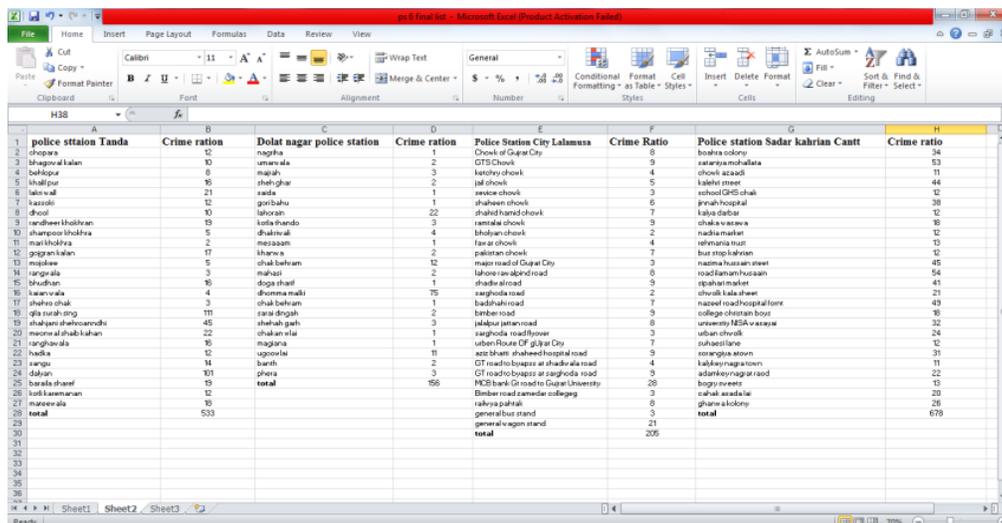


Fig.3: Crime Collection

## Data Marking

Step 1:

Map of District Gujrat map on Google earth

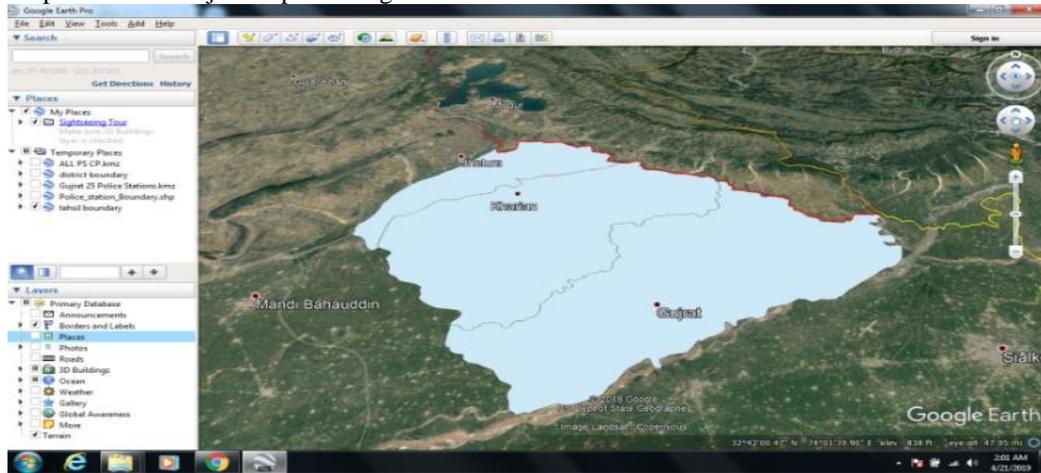


Fig 4: Gujrat map on Google earth

Step 2:

Showing police station boundary on a Google earth map

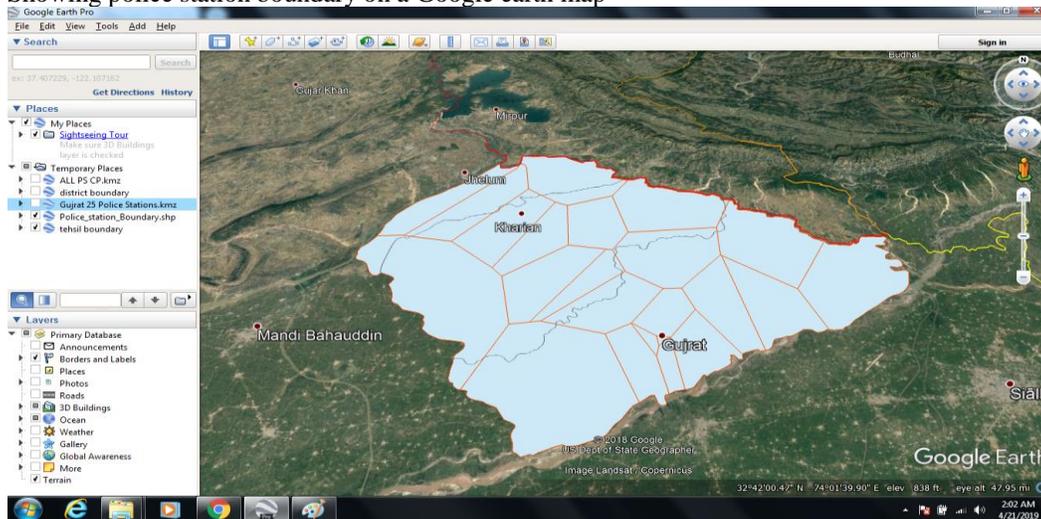


Fig 5: Police station boundary on a Google earth map



Step 5:

Converting google earth KML file for ArcGIS.

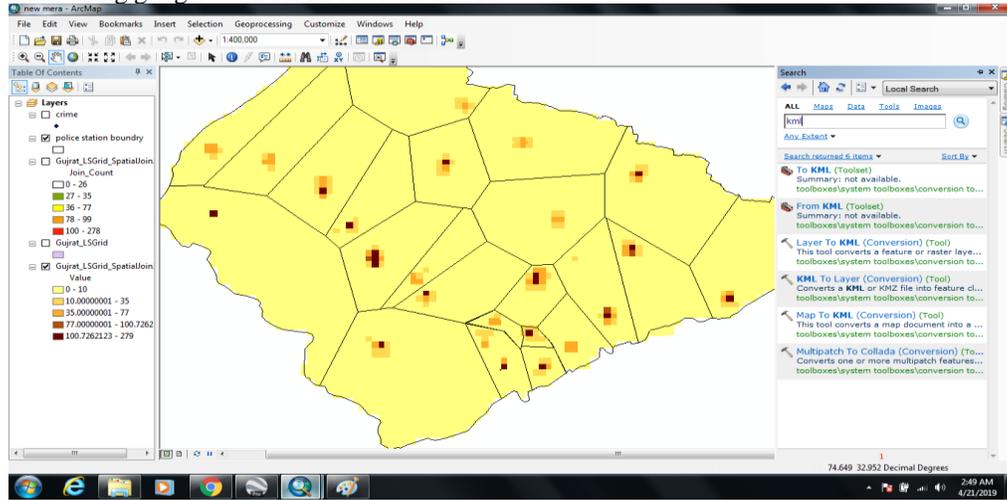


Fig 8: Converting google earth KML file for ArcGIS

Step 6:

After conversion into ArcGIS format for results.

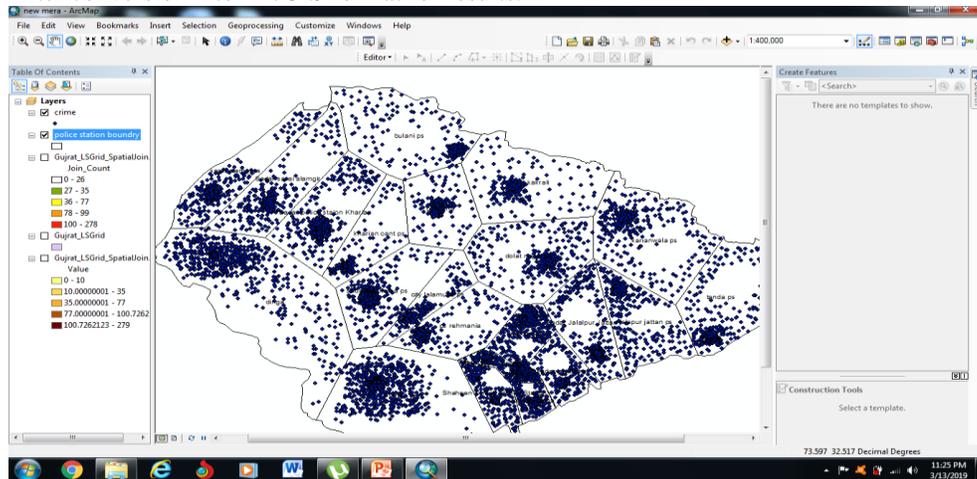


Fig 9: Point Conversion into ArcGIS format

Step 7:

Applied Grid 1skm by 1sqKm on the Gujrat District Area

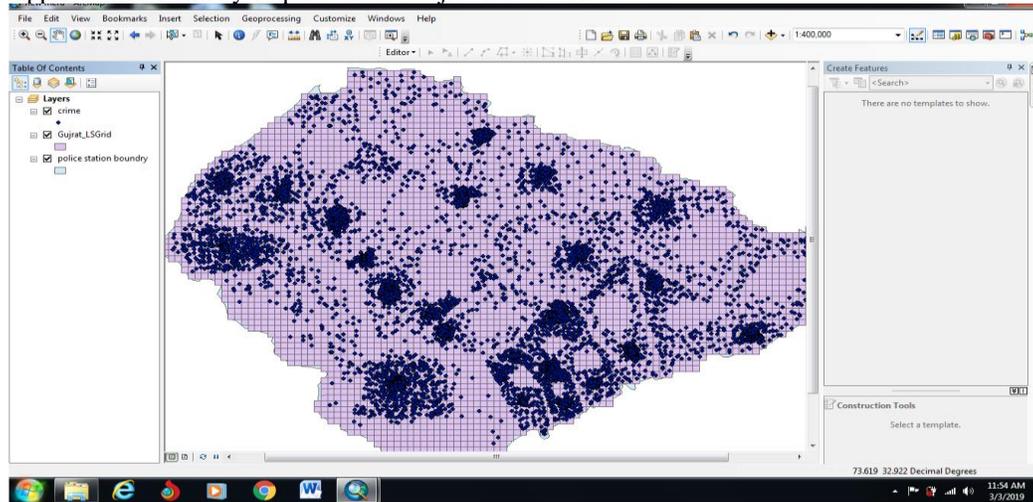


Fig 10: Applied Grid 1skm by 1sqKm

Step 8:

Applied crimes spatially join with grid.

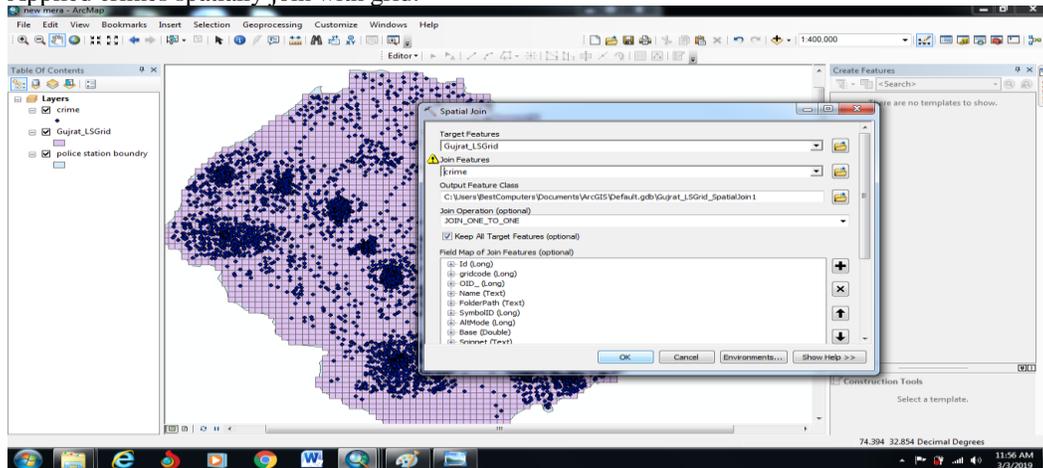


Fig 11: Crimes spatially join with grid

Step 9:

Showing high crime areas result.

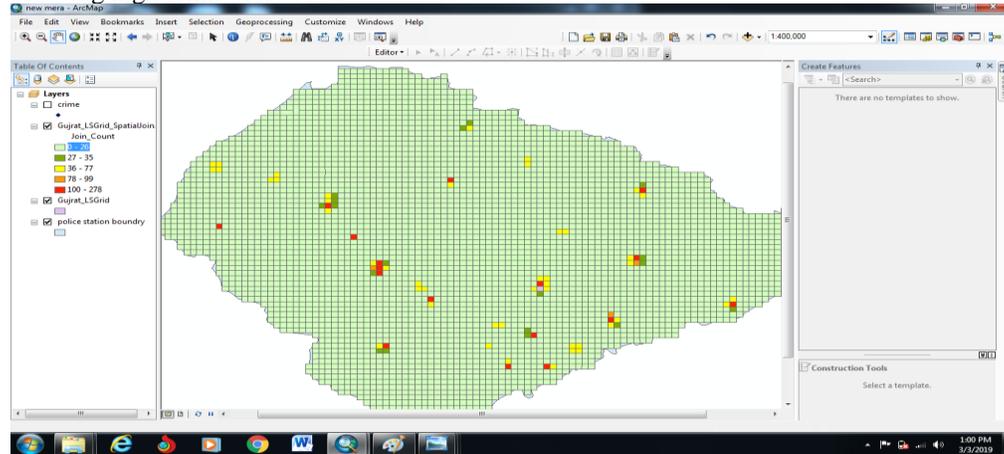


Fig 12: High Crime Areas

Step 10:

Conversion of Grid to Raster.

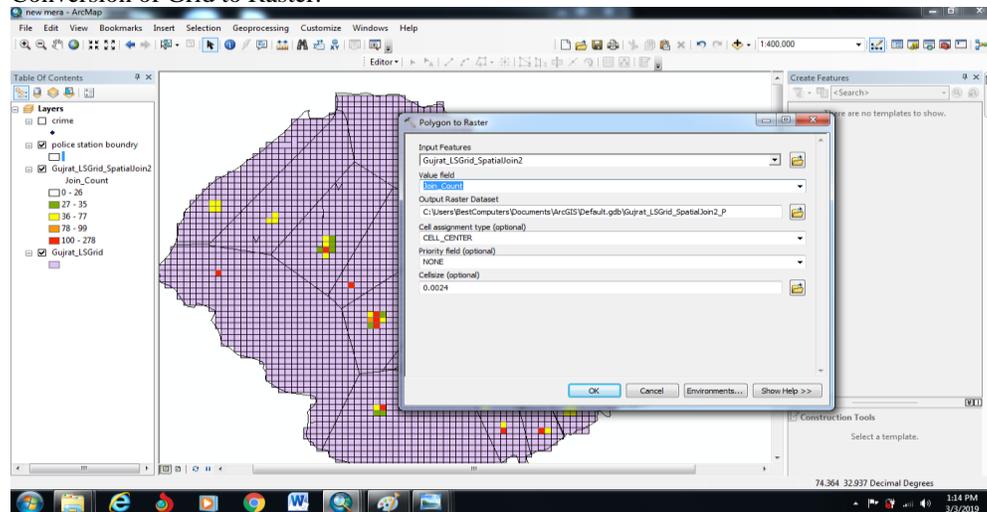


Fig 13: Conversion of Grid to Raster

Step 11:

Setting crimes classification into 5 classes with crime range we are showing crime range in figure but we can change into percentage.

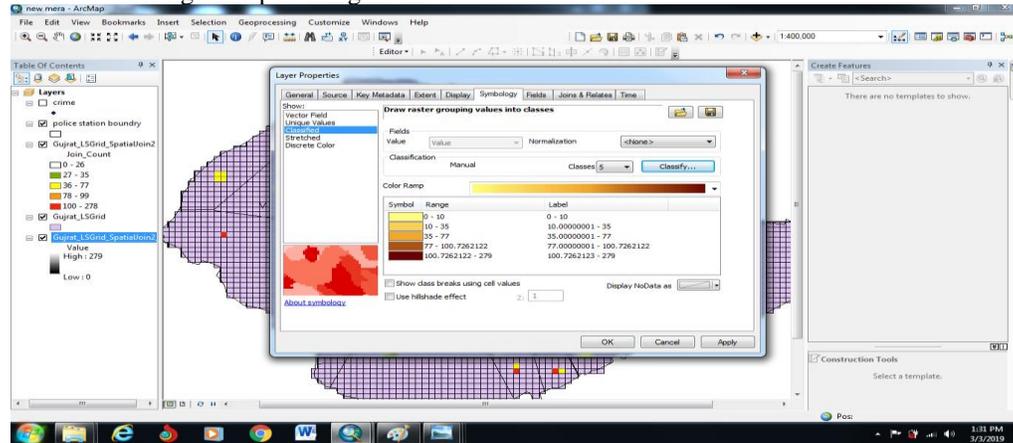


Fig 14: classes with crime range

Step 12:

Result Shows Crime Distribution  
(Safe Zone, Low Crime, Medium Crime, High Crime & Dangerous Area)

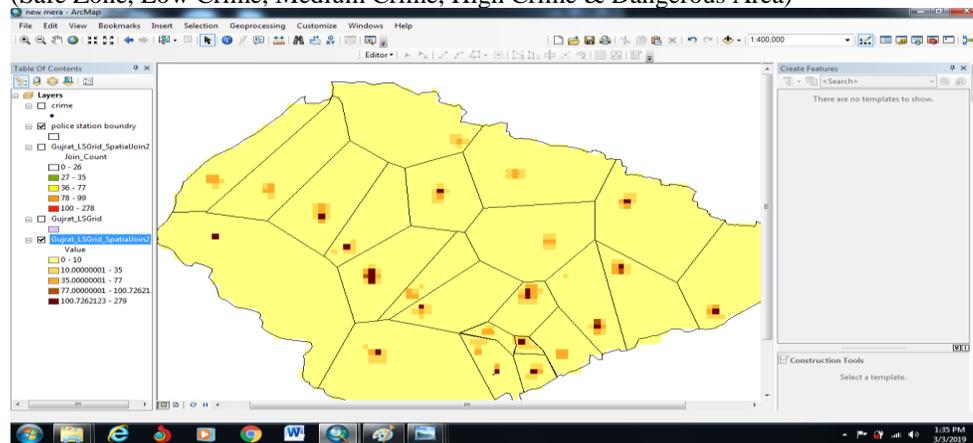


Fig 15: Crime Zones

Step 13:

Showing overall District Gujrat Crime result of all police stations.

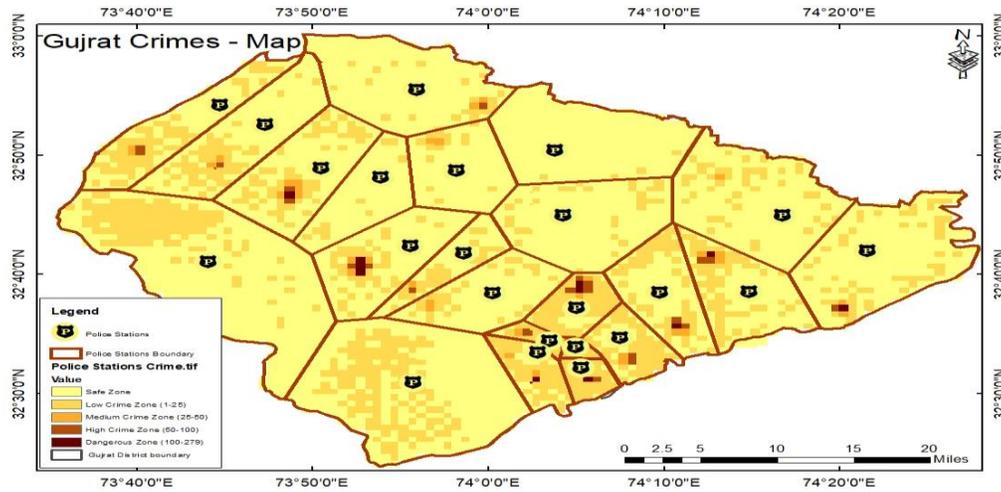


Fig 16: All Police Stations Results

7. Conclusion and Future Work

It is concluded from this learning that crime is considerate and deadly hassles in Gujrat. Diverse criminals are approximately stated from every corner of Gujrat at a dangerous charge, but criminology of this area exists in all the times and among all heads of the region to vicinity basis. The criminal entities have a superb dating with the attached landscape, social and economic structure that's located in this study so which will recognize the crime. The Study indicates that function of geography facilitates in simplifying the crime scenes. Positive space with respect to geographical elements and their relationship with criminology enables in information gathering in the form of the crime pattern called crime patterns in GIS. Gujrat is an important metropolitan in the region of Punjab so it has different geographical, economic, social, cultural, religious and political dimensions which are all taken an important part of each crime control and crime prevention system.

This study presents one of the most vital positions the first GIS based Jurisdiction map of Gujrat police stations. A novel and advance

surveillance versioning of Gujrat crime patrolling which is connected with via GPS (Global Positioning System). Our GPS with secondary source-based survey showed in this study that GIS primarily based map of police stations of Gujrat is also developed which is protecting relevant information of police stations. It is the first time in Gujrat that GPS with secondary survey is confirmed for acquiring the locations of criminal activities. Until this study, no such action was determined to be taken in Gujrat while such frameworks are employed and incorporated to detect and prevent major sort of crimes.

Whatever techniques make sense for you, concerning and analyzing GIS into crime mapping and protection is two times a hit preference as your development, your personal career must make substantial additions for social freedom and order. Think of it as advantages for one battle. GIS is used to measure and analyze crime rate with respect to spatial and temporal localities are different in geographical areas with respect to timings. These entities alter with the passage of time. The area, time zone, law, criminals, target or victim are elements of GIS based crime

patrolling system which are observed in geographical space. The GIS based strategy suggests law to interrupt crook to dedicate and target no longer homogeneous everywhere on the surface as a way to admire the change in location of criminal mindset. GIS apprehends the character of criminology using surrounding geography, demography and socioeconomic structure and these methods will take from the roots In-spite of that it will become a stable shaft. It is better to compare

past and present variations of crimes in Gujrat that are fantastically coping of crimes in Gujrat.

- Temporal analysis of Data
- Collect data with categories and types
- Every crime must be open in GIS form

## REFERENCES

- Remedies. IJMSSS, Kano State, Nigeria, vol. 1, no. 1, 2014
- [10] J. Shekwo, "Juvenile Delinquency in Mararaba, Karu L.G.A. of Nasarawa State, Nigeria", *International Journal of Sociology, Management & Security Studies*, Maiden Edition, Kano, Nigeria, 2013
- [11] S.A. Yelwa & Y. Bello, "Complimenting GIS and Cluster Analysis in Assessing Property Crime in Katsina State, Nigeria", *American International Journal of Contemporary Research*, Vol. 2, no. 7, 2012
- [12] S. Pires "crime mapping and analyses news", *International Journal of Science and Technology*, vol. 4, no. 5, pp. 1-30, 2012.
- [13] M.A.P chamikara., "GIS in crime analysis", *International Journal of Science and Technology*, vol. 3, no. 6, pp. 3, 2014.
- [14] J. Bueermann., "Crime analysis", *Journal of Environment and Earth Science*, vol. 2, no. 3, pp. 1-6, 2012.
- [15] M.brokmaan et al., "Crime Mapping and Analysis in the Dansoman Police Subdivision", *Journal of Environment and Earth Science*, vol. 4, no. 3, pp. 1-11, 2014.
- [16] C.D.J beaty., "GIS for Crime Analysis, Law Enforcement, GIS and Public Safety", *Journal of Environment and Earth Science*, vol. 4, no. 3, pp. 1-17, 2012.
- [17] T.Fransic et al., "Crime Mapping in Nigeria", scrip.org. [Online]. Available: <http://www.scrip.org/journal/PaperInformation.aspx?PaperID=50296>. [Accessed: 20- Nov- 2016].
- [18] j.corso et al., "Toward Predictive Crime Analysis via Social Media, Big Data, and GIS", *Journal of Environment and Earth Science*, vol. 2, no. 3, pp. 1-6, 2015.
- [1] T.F. Balogun et al., "Crime Mapping in Nigeria using GIS", *Journal of Geographic Information System*, vol. 6, no. 5, pp. 453- 466, 2014.
- [2] S.M. Ansari and Dr. K.V. Kale, "Methods of Crime Analysis using GIS", *International Journal of Scientific and Engineering Research*, vol. 5, no. 12, pp. 1330- 1336, 2014.
- [3] U.S. Usman, "The Role of Geographic Information System (GIS) in Effective Control of Terrorism in Nigeria", *International Journal of Economics, Commerce and Management*, vol. 3, no. 4, pp. 1- 9, 2015.
- [4] F. Fajemirokun et al., "A GIS Approach to Crime Mapping and Management in Nigeria: A Case Study of Victoria Island Lagos", Nigeria.
- [5] N. Levine, "Crime Mapping and the Crimestat Program", Wiley Online Library. [Online]. Available: <http://onlinelibrary.wiley.com/doi/10.1111/j.0016-7363.2005.00673.x/full>. [Accessed: 22- Nov- 2016].
- [6] A. Ahmed & R. Saliha, "spatiotemporal pattern of crime using GIS approach in Dala L.G.A. Kano state, Nigeria", *American Journal of Engineering Research*, vol. 2, no. 3, 2013.
- [7] G.O. Igbaekemen & S.U. Umar, "A Purview into the historical Development of Terrorism in Nigeria", *Journal of Developing country Studies*, vol. 4, no. 14, 2014.
- [8] O. James, "GIS and Crime Management in Nigeria", *International Journal of Sociology, Management and Security Studies*, Maiden Edition, Kano, Nigeria, 2013
- [9] F. Onu, "Corrupt Practices in Nigeria Society", A Search For Causes and

- [19] S.Muhammad et al., "Mapping and Analysis of Crime in Aurangabad City using GIS", IOSR Journal of Computer Engineering (IOSR-JCE), vol. 2, no. 4, pp. 67-76, 2014.
- [20] F.Wang., "Why Police and Policing need GIS", Journal of Environment and Earth Science, vol. 2, no. 4, pp. 67-76, 2012.
- [21] T.Balogan et al., "Crime Mapping in Nigeria Using GIS", Journal of Geographic Information System, vol.6, no. 4, pp. 453-466, 2014.
- [22] J. H. Davis and J. R. Cogdell, "Crime in GIS," Elect. Eng. Res. Lab., Univ. Texas, Austin, Tech. Memo. NGL-006-69-3, Nov. 15, 1987.
- [23] R. E. Haskell and C. T. Case, "Hotspot in Crime mapping," USAF Cambridge Res. Labs., Cambridge, MA, Rep. ARCRL-66-234 (II), 1994, vol. 2.
- [24] P. Diament and W. L. Lupatkin, "GIS in Crime Mapping," Dept. Elect. Eng., New York, Sci. Rep. 85, Aug. 1991
- [25] Crime with GIS, 3rd ed., Western Electric Co., Winston-Salem, NC, 2010, pp. 44-60.
- [26] GIS in Crime mapping., 1st ed., Spencer Chainey, Jerry Ratcliffe., 2010, pp.1-421.
- [27] "Hartsfield-Jackson Atlanta International Airport," [Online]. Available: [Http://www.esri.com/software/arcgis/arcgisonline](http://www.esri.com/software/arcgis/arcgisonline). Accessed: Nov. 25, 2016.
- [28] R. Gupta, Et Al., "Application of GIS in Crime Analysis: A Gateway to Safe City", 14th International Conference and Exhibition on Geospatial Information Technology and Applications. Vol.1, No.6, 2014.
- [29] Ferreria, Joao, "GIS For Crime Analysis: Geography For Predictive Models", Electronic Journal Information Systems Evolution. Vol.1, no.15 2012.
- [30] [Online]. Available. [Http://Link.Springer.Com/Chapter/10.1007%2f978-3-319-27793-6\\_1](http://link.springer.com/chapter/10.1007%2f978-3-319-27793-6_1), Accessed: Dec. 1, 2016.
- [31] Courso, Leroy, "Towards Predictive Crime Analysis via Social Media, Big Data, And GIS", Conference, Vol.1, no.6, 2015.
- [32] W. Fan, Y. Chen, "Design And Study On Police", Intelligence Analysis System Based On GIS. Vol.1 No. 4, 2011.
- [33] [Online]. Available. [Www.Slideshare.Net/Osokop/Fundamentals-of-crime-mapping1](http://www.slideshare.net/Osokop/Fundamentals-of-crime-mapping-1), Accessed: Dec. 1, 2016.
- [34] J.M. Caplan, "Mapping the spatial influence of crime correlates: A comparison of operationalization schemes and implications for crime analysis and criminal justice practice. Cityscape", Vol. 13, no. 3, pp. 57 – 83, 2011.
- [35] J. M. Caplan et al., Risk Terrain Modeling Diagnostics Utility User Manual (Version 1.0). Newark, NJ: Rutgers Center on Public Security. Retrieved March 1, 2014 from <http://rutgerscps.org/software>
- [36] M. Averdijk & H. Elffers, "The discrepancy between survey – based victim accounts and police reports revisited", International Review of Victimology, Vol. 18, no. 2, pp. 91 – 107, 2012
- [37] Katcrime, "Rising wave of criminality in Katsina". [Online]. Available: <http://www.nigeriaa2z.com/2012/04/08/rising-wave-of-criminality-in-katsina/>. [Accessed: 24- Dec- 2016].
- [38] L. W. Kennedy et al., "A Primer on the Spatial Dynamics of Crime Emergence and Persistence." (2012), Retrieved March 1, 2014 from <http://www.rutgerscps.org/PrimerOnCrimeBook.html>.
- [39] J.E. Gyong, "Criminal Victimization and the Reporting of Crime in Kaduna State: Towards Integrating the Victim of Crime into Criminological Discourse", Current Research Journal of Social Sciences, Vol. 2, no. 5, pp. 288 – 295, 2010
- [40] C. Dempsey, "GIS 101: Learn About GIS ~ GIS Lounge", GIS Lounge. [Online]. Available: <https://www.gislounge.com/gis-essentials/>. [Accessed: 12- Dec- 2016].
- [41] "Applications of GIS in Crime Mapping - ESDS Official Knowledgebase", ESDS Official Knowledgebase. [Online]. Available: <https://www.esds.co.in/kb/applications-of-gis-in-crime-mapping/>. [Accessed: 16- Nov- 2016].
- [42] A. Hiropoulos and J. Porter, "Visualising property crime in Gauteng: Applying GIS to crime pattern theory", South African Crime Quarterly, no. 47, 2016.
- [43] "Crime Mapping (continued) - How GIS Is Used in Law Enforcement", Ncjrs.gov. [Online]. Available:

- [https://www.ncjrs.gov/ovc\\_archives/reports/geoinfosys2003/cm3b.html](https://www.ncjrs.gov/ovc_archives/reports/geoinfosys2003/cm3b.html). [Accessed: 26- Nov- 2016].
- [44] "The Impact of GIS Technology - Nov 30, 2016", Journals.sagepub.com. [Online]. Available: <http://journals.sagepub.com/doi/abs/10.1068/b250681>. [Accessed: 20- Dec- 2016].
- [45] S. Jews"essays on geographic",International Journal of Science and Technology, vol. 3, no. 4, pp. 1-12 , 2012.
- [46] M.luciman., "using hotspot in GIS", International Journal of Science and Technology, vol. 2, no. 5, pp. 5 , 2014.
- [47] J.lexopiya., " crime act control using GIS", Journal of Environment and Earth Science, vol. 1, no. 2, pp. 1-3 , 2012.
- [48] S.ruceli et al., " Crime Mapping and Analysis", Journal of Environment and Earth Science, vol. 2, no. 2, pp. 1-11,2012.
- [49] C.Axeaty., "GIS and Public Safety", Journal of Environment and Earth Science, vol. 3, no. 2, pp. 1-10,2013.
- [50] I.crispo et al., " Big Data, and GIS", Journal of Environment and Earth Science, vol. 3, no. 4, pp. 1-4 , 2014.
- [51] S.Frezonet al., " Mapping and Analysis of crime", IOSR Journal of Computer Engineering (IOSR-JCE), vol. 1, no. 3, pp. 61-70 , 2012.
- [52] P.Wrex., " Policing need GIS", Journal of Environment and Earth Science, vol. 1, no.3, pp. 61-65 , 2013.
- [53] Y.Manishia., "Crime effects", International Journal of Science and Technology, vol. 1, no. 2, pp.3 , 2012.

# Implementation of Kanban Techniques in Software Development Process: An Empirical Study Based on Benefits and Challenges

Muhammad Noman Riaz<sup>1</sup>

---

## Abstract:

It has been observed that there is an increasing trend of implementing Lean Techniques as a whole in general and Kanban in particular in the software development process all across the globe to reap the proclaimed advantages of these techniques as documented in the available literature and verified by the Lean practitioners. The main goal of this paper is to highlight the present state of implementation and usage of Kanban techniques in the software development process, its obtained and to-be-obtained benefits, challenges that are being faced by the software development companies and the reason(s) of motivation that compels the organization to adopt Kanban technique. Furthermore, the solutions for challenges identified during the study will also be suggested. In order to achieve this, an empirical study has been conducted encompassing a survey followed by complementing thematic interview sessions with 241 employees of 67 software companies of Pakistan. The companies involved in the empirical study have reasonable years of experience in practicing Agile and Lean techniques. Generally, the obtained results depict that the usage of Kanban techniques is rather positive and appreciative; however, the challenges that are being faced in Kanban adoption are lack of specialized training and experience pertaining to its usage, and a too conventional organizational culture.

**Keywords:** Agile, Kanban, Lean, Muda, Software Development

---

## 1. Introduction

Nowadays, the Agile and Lean techniques are being extensively and successfully implemented in different industries across the globe. The Lean manufacturing / maintenance techniques were developed and implemented in manufacturing industries in different parts of the world [1] and subsequently found effective in producing the desired results. Later on, the Lean techniques were also adopted by the software development firms.

The primary function of Lean technique is to eliminate the waste or non-value-added activity from the line of process before it hampers the development / production. The authors of [1] transformed the Lean techniques for appropriate usage in software development realm. They proposed the following six set of principles:

- Creation of Quality
- Creation of Knowledge
- Commitment Deferment

---

<sup>1</sup> Department of Computer Science, Virtual University of Pakistan Lahore, Pakistan  
Corresponding Author: MS150400221@vu.edu.pk

- Fast Delivery
- People Respect
- Optimization of the whole

In the context of software development any activity that does not contribute anything significant is termed as “Muda” or “Waste”. For example, additional required processes, partially completed tasks, additional unwanted features, defects and units, untimely switching of tasks are considered as “Wastes” [1].

Kanban is one of the ways of implementing Lean technique that has been in usage in the management of production operations at Toyota Company for decades [2]. Kanban is one of the most recent additions in Lean and Agile software development methodologies. During the last few years the Kanban has become increasingly popular in the software development. A very powerful practitioners-driven movement emerged in its support [3], [4]. Presently, the Kanban approaches are confidently used by the software engineering practitioners to complement Agile, Scrum and other methodologies in software development processes.

However, despite the increasing popularity and increasing usage in software development, the present literature has failed to highlight the Kanban techniques appropriately in the context of software development. Only, a few numbers of studies on the usage of Kanban, how it can be effectively practiced, and its potential impact on the software development have been published [5]. With the aim of providing the latest status of Kanban usage and its impact on the software development and subsequent reputation of software companies, an empirical study has been conducted. The study encompasses 241 software engineering professionals belong to 67 software companies registered with Pakistan Software Export Board (PSEB). The survey has been conducted followed by interviews with high profile software engineering professionals working in Pakistan. The interviews addressed the following questions:

- What actually the benefits the Kanban technique is capable of providing?
- What are the potential challenges that the organization(s) may face in its adoption?
- What are appropriate solutions in addressing the Kanban usage challenges?

The rest of the paper is organized as follows: Section II presents the literature review on the usage of Kanban technique. Section III describes the research settings that include the data collection process as well as the design of survey and complementing interviews. Section IV elaborates the obtained results and compares them with the findings and results of previously conducted research. The last section, Section V concludes the paper while summarizing the obtained results and discussing the limitations of the conducted research.

## 2. Background of Kanban

The Lean technique, Kanban, was first introduced in 1950s by one of the senior executives, Mr. Taichi Ohno, of Toyota Company in the Japanese manufacturing industry. Literally, the term Kanban means the signboard or visualization of inventory items or system used in the scheduling system for Just in Time (JIT) production. The main reason behind the development of Kanban technique was to find and propose a system that guarantees the maintenance and improvement in the production system. Kanban is basically a flow driven mechanism for pull-driven JIT production, in that the upstream processing items / activities are triggered by the downstream process demand signals [2], [7]. The Kanban technique has been proven as production improvement technique and guarantees the smooth running of production system. The Kanban technique was successfully implemented and practiced in Toyota Production System (TPS) and the main reason of implementing Kanban at TPS was to execute the entire range of Lean manufacturing practices in length and breadth

of TPS as Lean is more than the Kanban technique [7], [8], [9].

In today’s industry of software development, the appreciation of Lean and Kanban is hugely driven by practitioners’ books [1], [6], [10]. Both the Lean and Kanban principles seem to immensely overlapping and hence depict the same foundation. The Table 1 below depicts the Lean principles of software development [1], [20] and Kanban principles [10], which are familiar to the Agile community.

**Table I.** Lean and Kanban Principles

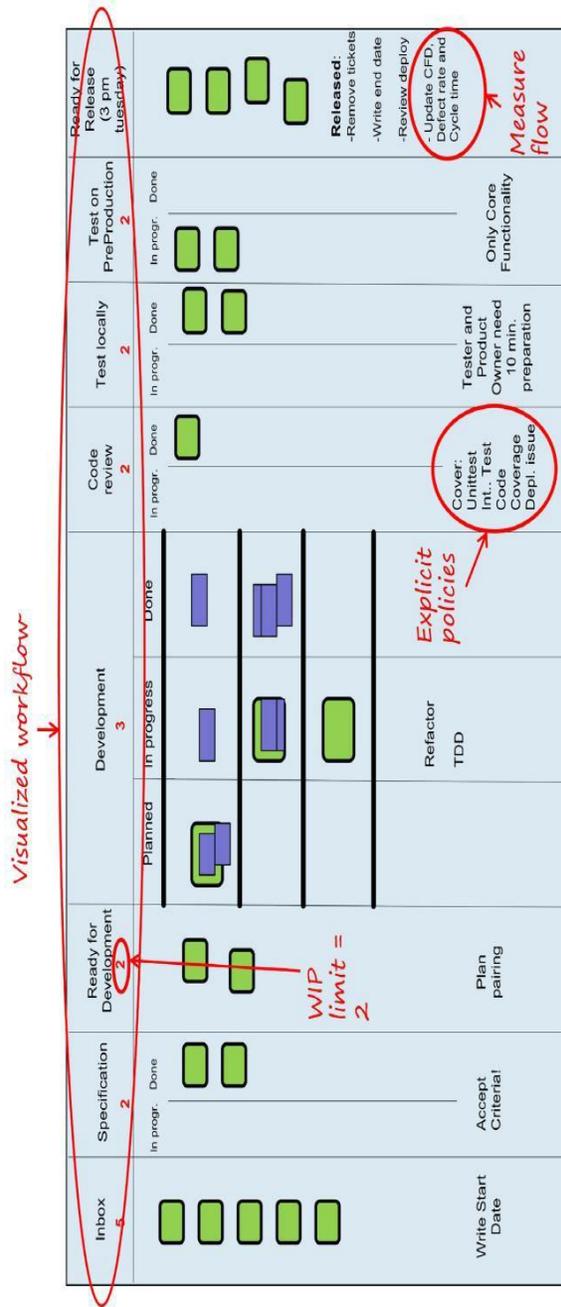
Lean Software Development Principles [20]	Kanban Principles [20]
Waste Elimination	Work flow Visualization
Build Quality in	Limit Work in Progress
Knowledge Creation	Measurement and Management of Flow
Commitment Deferment	Explicit Process Policy Making
Fast Delivery	Improvement via Collaboration
People Respect	
Optimize the Entire	

In the year 2004, a software development professional, David J. Anderson, proposed an idea of implementing a Kanban technique in the software development when he was involved in assisting an IT team at Microsoft which was operating at below par level [10]. While assisting the project, David J. Anderson [10], introduced and urged upon the usage of Kanban technique to the members of IT team and asked them to visualize their activities and put limits on their Work in Progress (WIP). The main objective behind visualizing the activities and put certain limits on WIPs was the identification of constraints in the process (es) and to remain focus on one single activity / item at a time. As a result, the “Pull”

approach has promoted. In a conventional software development process, each team member or a small group of members have to complete the tasks in stipulated time and forward it the next person or group in line i.e. the items are “pushed” towards the next responsible person / team who are directed to complete the forwarded / pushed tasks as early as possible and as maximum as possible. In fact this form of working style is basically a “chain” type in which the items are forwarded to next level in sequence i.e. from development to testing. This traditional style of software development has an inherent tendency of injecting delays in the entire software development process especially when the next responsible individual or group is overwhelmed with amount of work. However, the Kanban technique works in the other way. Rather than promoting the “push” approach i.e. pushing the items in sequence to the next level it promotes a “pull” concept. In Kanban approach, each team member/group works on one single item at a time and when that in hand task is completed only then the individual / group “pull” another item / task to work on.

In short, the Kanban technique is responsible to provide the visibility of software development process, disseminate priorities and identify bottlenecks [5]. This guarantees the constant distribution of work items to the stakeholders / customers. In this the developers only involved in those small items which are worthy for the customer [6]. The figure 1 below depicts a typical structure and principles of Kanban board in practice.

The incorporation of Kanban technique in software development process assists the software developers in highlighting their current tasks, the current work flow, limiting the WIP at each stage of work flow, and in measuring the cycle time [11]. The primary objective of Kanban is to remain focus on the present work flow and ensure the absence of essential iterations.



**Fig 1.** Kanban Structure and Principles Board in Practice [12], [20]

The practice of Kanban usage in manufacturing industries has yielded many fruitful results and advantages. The strong and successful historical data of Kanban implementation has urged the software developers to incorporate Kanban technique in software development and as a result the usage of Kanban technique has won the confidence of software development professionals and has gained well-built practitioners-driven support for its adaptation.

The Kanban technique is software industry was proliferated with boom after the publication of two well-known books [10], [12]. One of these books was authored by David J. Anderson with a title of “Kanban” [10]. In this book the author elaborated the concepts of Kanban in the development of software and systems. The other book was authored was Corey Lada and entitled, “Scrumban” [12], in the book the author discussed the fusion of Kanban and Scrum techniques.

The literature review [6] conducted in Kanban techniques few years ago revealed several challenges and benefits in the adaptation of Kanban technique in software development industry. Few of the benefits that the implementation and practice of Kanban technique guarantees include: the Kanban technique makes easier the understanding of whole development process, helps in augmenting the level of coordination and confidence among team members and project / production stakeholders, and facilitates the development team in understanding the requirements and satisfaction of the customers [14], [15], [16]. According the study [17], It was revealed that the Kanban technique has significantly improved the performance of the development teams, increased the customer satisfaction level and reduced the number of defects in the product. As compare to the previously used Agile method the lead-time of software development increased by 37%,

delivery consistent rate increased by 47% and the number of customers reported defects have fallen by 24% [17]. Due to the WIP limits, the items that are of higher priority and need to be executed / developed earlier than the other items approach are pulled to optimize the value, that resulted in enhanced customer satisfaction level [14], [15], [17]. Another advantage of Kanban technique is that it does not encourage the cramming of user requirements because the Kanban technique does not follow the fixed plan [21].

According to the results of [22], to work more effectively the Kanban technique needs to be implemented / practiced with other Agile techniques. But the fusion of Agile and Kanban techniques has a tendency of generating problems for some teams [18]. To encourage and motivate the developers to use Kanban has been a challenging task mainly due to organizational culture and stickiness and reluctance of software developers who are more comfortable and experienced in other similar development methodologies [16], [18], [23]. Besides this, the lack of specialized skills and adequate training, and misapprehension of core principles have significantly contributed in adoption Kanban technique [14], [16], [18], [24], [25].

### 3. Research Settings

The empirical data for study were collected from the professionals of volunteered participating companies. This empirical study was successfully conducted in two different stages. In the first stage of the survey, the usage of Kanban technique was carried out among the participating companies. During the survey the Internet Survey Tool was utilized. During the survey the questions were encompassing the usage of Kanban, the benefits achieved and associated with the usage of Kanban, motivation and reason to implement Kanban in the development and the

challenges that create obstacle in the adoption of Kanban technique.

During the second stage of the survey, the thematic semi-structured interview sessions of 65-85 minutes duration were conducted with the volunteered survey respondents. These interview sessions roamed around the benefits that the practicing organizations are reaping and the challenges they are facing in the adoption of Kanban. The detailed discussions were carried out with the respondents in which the probable solutions of addressing the challenges of Kanban adoption were shared vividly by the respondents.

In the first stage of the survey, an Internet Survey Tool was used and the requests were sent to the hand-picked professionals of 67 software companies operating in Pakistan and registered with Pakistan Software Export Board (PSEB). The reason for sending the emails to the selected software professionals was to identify and rate the significance of Kanban usage, the motivation behind its usage, and the benefits achieved and the challenges faced in Kanban adoption. A five-point Likert scale was used during the survey to rate the received responses. During the survey a total of 241 persons representing 67 software companies responded to the survey questionnaire.

During the second stage of the survey, a total of 67 managerial level professionals of 67 software companies were individually interviewed. The primary reason of these interview sessions was to complement the first stage responses of the volunteers on Kanban usage. The duration of interview sessions was between 65 and 95 minutes with an average time of 75 minutes. All the interview sessions were recorded and transcribe by the interviewers, and to ensure consistency the transcripts were thoroughly checked by the individual interview.

#### 4. Results

In this section, the results obtained after the conduct of survey and semi-structured interview sessions are being presented and subsequently compared with the results of earlier conducted studies on Kanban usage in software industry.

##### A. Survey Results

The Table II and Table III below depict the appointments / positions and the level of experience the volunteered respondents of the survey possess in software development.

**Table II.** Interviewees background

Appointments	n
President / CEO	35
CTO / VP / GM / Prog Manager	45
Product Owner / Product Manager	55
Scrum Trainer / Agile Coach	60
Software Analyst / Designer / Developer	25
Coach / Trainer / Consultant	15
Software Programmer	06
Total	241

**Table III.** Software development experience

No of Years' Experience	n	%
1 - 5	06	2.5
6 - 10	100	41.5
11 - 15	55	23
15 - 20	45	19
More than 20	35	14
Total	241	100

The volunteered respondents of the survey are working on different managerial levels in their respective organizations. Most of the respondents are working at mid-level management (Program Managers, Project Managers, Software Analysts, Agile Coaches etc.) and 86% of them possess more than 10

years of software development experience. The typical project team sizes in practice in the participating organizations are depicted below in Table IV.

**Table IV.** Teams size

Teams Size	n	%
1 - 4	17	25
5 - 8	15	23
9- 12	23	34
13-15	10	15
15 and greater	2	3
Total	67	100

From the survey it was noticed that most of the project teams comprise of less than 10 professionals only. There are only two software development companies that have teams comprising more 13 professionals.

While performing the data analysis of the survey it was revealed by approximately 67% of the respondents that a Kanban technique is being used in their organization for managing the software development processes. Most of the Kanban practicing organizations that are included in the survey had been practicing Kanban for more than two years. Also, the software development professionals working in these organizations have considered themselves proficient and expert in implementation of Kanban technique.

During the interview sessions with the volunteered respondents, several questions were asked that encompassed the motivational factors of using Kanban technique, benefits achieved by the practicing organizations and the challenges associated with the Kanban. The results are depicted in Table V-VII based on the responses of the subjects.

**Table V.** Motivational factors in using Kanban

Motivation for Selecting Kanban Method	Median	Mean
Improvement in Team Communication	5.4	6
Improvement in Development Flow	5.2	6
Reduction in Development Cycle Times & Time-to-Market	5.1	5
Productivity Enhancement	5.1	5
Creation of Transparency in Organization	4.8	5
Improvement in Service & Product Quality	4.8	5
Comprehension of Entire Value Stream	4.5	5
Improvement of Process Quality	4.5	5
Augmenting Satisfaction of Stakeholders'	4.3	5
Elimination of Excess Activities	4.3	4
Nurturing the Ability to Adapt to Changes in Business Arena	4.8	4
Reduction in Development Costs	3.9	4
Improvement in Organizational Learning	3.6	4
Management of Product Value	3.5	4
Increased Customer Understanding / Trust	3.6	3

The above table clearly depicts that the highest motivation factors in the adaptation of Kanban technique is the effective communication between the team members

working on a similar project as well between the teams working on different task of similar project. Also, the usage of Kanban technique encourages the reduction in product development life cycle and the time-to-market the product, helps in improving the flow of product development and bolsters the productivity of the organization [25]. Besides guaranteeing the aforementioned advantages the Kanban technique most importantly ensures the transparency within the organization. According to the research of the authors [11], the implementation of Kanban technique is highly motivated by its ease of adaptability; its visualization style of monitoring the progress creates easiness for the workers [26], and drives the project team members to effectively and efficiently communicate with each other during the project. As far as the software development is concerned the motivation factors are aligned with what Anderson elaborated [10], [27]. During the interview sessions the subjects / respondents were also asked to indicate the benefits achieved by their respective organizations. The achieved benefits of the organizations are presented below in Table VI.

**Table VI.** Benefits Achieved by Organizations After Using Kanban Method

Benefits Achieved	Median	Mean
Excellent Work Visibility	6	5.9
Better Transparency in Work	6	5.5
Better Communication between Team Members / Teams	5	5.3
Improved Development Flow Control	5	5.2
Improved Focus on Routine Work Activities	5	5.1

Improved Control on Work In Progress (WIP)	5	5.0
Improved Team Efficiency	5	4.9
Better Risk Management	5	4.7
Improvement in the Comprehension of Entire Development Process	5	4.6
Beneficial in Building Team Trust	5	4.6
Assists in Inevitability	5	4.5
Decrement in Switching of Environment	4	4.5
Improvement in Quality	4	4.1
Assistance in Strategic Decision Making for Organizational Leadership	4	3.7
Better Knowledge Sharing	5	5.2
Better Stakeholder Management	4	4.2
Improved Employee Motivation	5	4.8
Reduced Job Turn Over	5	5.1
Improved Inventory Management	4	4.6
Elimination of Overproduction	5	4.9
Decreased Risk of Inventory Devolution	4	4.6
High Level of Employee Empowerment	5	5.7
Better Quality Control	5	5.4
Smooth Development Flow	5	5.8

Improved Visibility of Routine Work, Enhanced Transparency pertaining to Team Communication and Better Control of Work Flow. All these common experienced benefits (except that of Better Risk Management and Better Knowledge Sharing among the team members / teams) are also highlighted in the literature [4], [6], [10], [12], [19], [27].

Moreover, during the interview sessions the respondents / subjects were asked to identify the challenges that they and their organizations are facing in the implementation of Kanban technique in true letter and spirit. The reasons the respondents identified are listed in order of significance in Table VII.

From the results it has been concluded that the main hurdle in the implementation of Kanban technique in software development is employees' lack of experience in practicing Kanban technique. The other identified challenges in the usage of Kanban are difficulty in prioritizing the workload / tasks of the ongoing projects and management of Work in Progress (WIP) limits. The traditional organizational culture and no clear road map for future development are also the major contributing factor in non-practicing of Kanban method. We can easily and confidently infer from the above discussion that all these challenges are simply associated with one problem: a scarcity of adequate knowledge and specialized training.

The Table VI above depicts that the most of the benefits achieved by the organizations are almost the same as revealed in the available literature except that of Better Risk Management and Better Knowledge Sharing among the team members / teams. The most common benefits experienced by the subjects / respondents were Better Knowledge Sharing, Better Knowledge Sharing,

**Table VII.** Kanban Implementation Challenges

Kanban Implementation Challenges	Median	Mean
Employees Lack of Experience in Practicing Kanban	5	5.0
Difficulty in Managing Work In Progress (WIP) Limits	5	4.9
Difficulty in Prioritizing Tasks	4	4.3
Traditional Organizational Culture	4	4.2
Dearth in Specialized Training & Knowledge	4	4.1
Lack of Vision in Product Development	4	4.0
Team Members Reluctant to Switch from Traditional Methods	4	3.9
Inefficiency in Decision Making	4	3.6
Absence of Developer/Customer Collaboration	4	3.5
Lack of Planning in Adoption of Kanban Technique	4	3.4
Absence of Sponsorship from Organizational Management	4	3.4
Reduction in Predictability	4	3.4
Team Members / Teams Feel Burden of Communication	4	3.3
Reluctance shown by Customers for Increased Communication	3	3.2
Kanban Method and Business Domain Incompatibility Issue	3	2.9

Reluctance in Adoption from Old Employees	3	2.7
Lack of Experience in Kanban Usage	3	2.9

### B. Results of Interview Sessions

In this sub-section the results obtained from the conducted interview sessions are elaborated. The obtained results have been categorized in three main themes: obtained benefits, challenges faced by the organizations, and the appropriate solutions in countering these Kanban implementation challenges.

#### 1) Obtained Benefits

Majority of the respondents communicated that the Kanban works extremely well within the teams of their organizations. During the interview sessions of the interviewees mentioned one of the benefits that he has experienced while practicing the Kanban is, *“One of the main benefits that we in our organization have experienced is that the customer feedback loop has enhanced”*.

In connection with the obtained survey results the respondents have also revealed the impact of Kanban usage in team work. One of the interviewees quite confidently revealed that, *“Kanban has brought life to our projects and relationships. Team members rather than focusing on the end results, we as team members, focused us on the ongoing processes. This helps us in finding the balance between the customer demand and our capabilities”*. Also, one of the interviewees elaborated that Kanban has benefited us in our *“Testing System”*, he said, *“A Kanban adoption has proved beneficial for us because it assists us in limiting the tasks after its release and we get some time to explore the mistakes we committed while performing these tasks. In this manner the things or difficulties that were unable to be found are being found easily and hence the number of*

*bugs per release has been decreased manifold”.*

Furthermore, all the interviewees were of the view that the usage of Kanban technique in the software development that it makes the team collaboration visible and even more amount of work can be completed collectively with less cost and difficulty. One of the respondents claimed, *“The use of Kanban technique helps in strengthen the bonding between the team members and hence helps in achieving the desired results easily”.* Another respondent revealed, *“A Kanban technique helps those workers who have trouble in performing their tasks in a way that other workers assist them collectively”.* Furthermore, it was identified by another interviewee, *“The team members collaboratively and collectively start finding where the things have been stuck and what would be the probable cause of this stagnation and make every endeavor to troubleshoot the obstacles that are present in the flow of working”.* Additionally, one interviewee mentioned, *“A Kanban technique ensures the organization of work with flexibility and efficiency”.*

## **2) Challenges Faced by the Organization in the Usage of Kanban Technique**

During the survey it was found out that the biggest challenge the organizations are facing the implementation of Kanban technique is that the employees are having no prior Kanban practicing experience, and that deficiency leads to other challenges that include difficulty in prioritizing the in-hand tasks and the management of WIP limits etc. In this context, one of the interviewees explained with heavy heart, *“Unfortunately, in our company, not many of the professional software engineers / developers are having prior working knowledge of Kanban technique. Also, those have the requisite skills and experience in the implementation of*

*Kanban also feels under confident in adopting the Kanban technique in the organization. Furthermore, there are people in our organization who are resistant to change and do not want the adaption of Kanban in software development”.*

Furthermore, another respondent revealed, *“Actually the upper management is reluctant to introduce Kanban in our organization. The main reason for discouraging the usage of Kanban is that upper management has no confidence in the abilities of Kanban. They are of the view that if the experiment of using Kanban fails then that will surely lead to loss of market share”.*

Another respondent reported, *“One of the biggest challenges that we are facing in the implementation of Kanban in our organization is that of a lack of knowledge and expertise of using Kanban in software development. Nobody is sure of using Kanban technique in software development both at team as well as a portfolio level. The employees usually ask a question, What is in it (Kanban technique) for us? And unfortunately, no one in our organization has answered this question with confidence”.*

Furthermore, besides requiring the adequate knowledge and specialized training, a positive approach or mind-set is an essential requirement in adopting and implementing any new concept. Without an adequate knowledge and experience it would be difficult to motivate the workforce for using the Kanban technique in their routine software development activities. This thing also compels the teams to revert to their previously adopting methodology / approach. In this context one of the respondents replied, *“It takes a lot of time and efforts to change the mature mind-set of staunch traditionalists”.*

## **3) Proposed Solutions to the Challenges**

During the interview sessions the respondents were asked about the possible and probable

solutions of the Kanban usage challenges. Based on the survey, the following solutions have been proposed by the respondents to mitigate the Kanban usage challenges.

- An adequate training and exposure to the usage of Kanban in software development must be provided to the software developers and managers
- It is recommended that the teams must allow to experiment or start a pilot project of small-scale to gain hands on experience in Kanban usage
- The upper management of the organization must take every possible step to change the mind-set of the employees. This will surely help in minimizing the opposition of Kanban usage
- Appropriate number of seminars must be conducted in regular intervals to promote the use of Kanban

## 5. Conclusion

During the study it was revealed that the main motivational factors that contribute in the usage of Kanban technique are; the effective communication between teams and team members, reduced time for the product to access the market, enhanced productivity, better risk management and knowledge sharing, and creation of transparency within the organization. Despite the limited scope of study, however, this study provides the valuable contemporary state of the usage of Kanban technique in software companies of Pakistan. This study significantly contributes in providing firsthand information of Kanban usage in software industry of Pakistan. Also, the ground realities and challenges that being faced by the software practitioners in Pakistan software industry have been bluntly and candidly discussed by the interviewees. Lastly, this study will provide a solid foundation for further exploring the challenges and their probable solutions in

implementing Kanban technique in software development companies of Pakistan.

## REFERENCES

- [1] M. Poppendieck and T. Poppendieck, *Lean Software Development: An Agile Toolkit*, Addison-Wesley Professional, 2003.
- [2] J. Liker, *The Toyota Way, USA*: McGraw-Hill, 2004.
- [3] K. Hiranabe, *Kanban applied to software development: From agile to lean*, InfoQ. [Online] Available from: <http://www.infoq.com/articles/hiranabe-lean-agile-kanban> 2014.05.04
- [4] A. Shalloway, B. Guy, and R. James Trott, *Lean-agile Software Development: Achieving Enterprise Agility*, Pearson Education, 2009.
- [5] Cloud Software Finland, *General Brochure*. [Online] Available from: <http://www.n4s.fi/en/>, <http://www.cloudsoftwareprogram.org/general-brochure> 2014.05.13
- [6] M. O. Ahmad, J. Markkula, and M. Oivo, "Kanban in Software Development: A Systematic Literature Review," 39th Euromicro Conference on Software Engineering and Advanced Applications (SEAA), 2013, pp. 9-16.
- [7] L. Chai, "E-based Inter-enterprise Supply Chain Kanban for Demand and Order Fulfilment Management," *International Conference on Emerging Technologies and Factory Automation*, 2008, pp. 33-35.
- [8] M. Becker and H. Szczerbicka, "Modeling and Optimization of Kanban Controlled Manufacturing Systems with GSPN including QN," *International Conference on Systems, Man, and Cybernetics*, 1998, vol. 1, pp. 570-575.
- [9] M. Ikonen and P. Abrahamsson, "Anticipating Success of a Business-Critical Software Project: A Comparative Case Study of Waterfall and Agile

- Approaches", In Proc. ICSOB, 2010, pp. 187-192.
- [10] D. Anderson, *Kanban – Successful Evolutionary Change for Your Technology Business*. Blue Hole Press, 2010.
- [11] H. Kniberg and M. Skarin, "Kanban and scrum – Making the most of both," *InfoQ*, 2010.
- [12] C. Ladas, *Scrumban – Essays on Kanban Systems for Lean Software Development*, Modus Cooperandi Press, 2009.
- [13] J. Boeg, *Priming Kanban, A 10 Step Guide to Optimizing Flow in Your Software Delivery System*, Trifork Agile Excellence Mini Book Series, 2<sup>nd</sup> ed., Denmark at Chronografisk A/S, 2012.
- [14] M. Senapathi, P. Middleton, and G. Evans, "Factors Affecting Effectiveness of Agile Usage – Insights from the BBC Worldwide Case Study," In Proc. XP, 2011, pp. 132-145.
- [15] M. Ikonen, E. Pirinen, F. Fagerholm, P. Kettunen, and P. Abrahamsson, "On the Impact of Kanban on Software Project Work: An Empirical Case Study Investigation," In Proceedings of ICECCS, 2011, pp. 305-314.
- [16] N. Nikitina and M. Kajko-Mattsson, "Developer-driven big-bang process transition from Scrum to Kanban," In Proceedings of ICSSP, 2011, pp. 159-168.
- [17] P. Middleton, and D. Joyce, "Lean software management: BBC Worldwide case study," *IEEE Transactions on Engineering Management*, 2012, vol. 59(1), pp. 20-32.
- [18] V. G. Stray, N. B. Moe, and T. Dingsøy, "Challenges to Teamwork: A Multiple Case Study of Two Agile Teams," In Proceedings of XP, 2011, pp. 146-161.
- [19] M. O. Ahmad, K. Liukkunen, J. Markkula, "Student perceptions and attitudes towards the software factory as a learning environment," In Proceedings of IEEE EDUCON, 2014, pp. 422,428.
- [20] Ahmad, M. O., Markkula, J., Oivo, M., & Kuvaja, P. (2014, October). Usage of Kanban in software companies: an empirical study on motivation, benefits and challenges. In Proceedings of 9th International Conference on Software Engineering Advances.
- [21] Kirovska, N., & Koceski, S. (2015). Usage of Kanban methodology at software development teams. *Journal of Applied Economics and Business*, 3(3), 25-34.
- [22] Hofmann, C., Lauber, S., Haefner, B., & Lanza, G. (2018). Development of an agile development method based on Kanban for distributed part-time teams and an introduction framework. *Procedia Manufacturing*, 23, 45-50.
- [23] Ahmad, M. O., Dennehy, D., Conboy, K., & Oivo, M. (2018). Kanban in software engineering: A systematic mapping study. *Journal of Systems and Software*, 137, 96-113.
- [24] Tanner, M., & Dauane, M. (2017). THE USE OF KANBAN TO ALLEVIATE COLLABORATION AND COMMUNICATION CHALLENGES OF GLOBAL SOFTWARE DEVELOPMENT. *Issues in Informing Science & Information Technology*, 14.
- [25] Hofmann, C., Lauber, S., Haefner, B., & Lanza, G. (2018). Development of an agile development method based on Kanban for distributed part-time teams and an introduction framework. *Procedia Manufacturing*, 23, 45-50.
- [26] Ahmad, M. O., Dennehy, D., Conboy, K., & Oivo, M. (2018). Kanban in software engineering: A systematic mapping study. *Journal of Systems and Software*, 137, 96-113.
- [27] Lei, H., Ganjeizadeh, F., Jayachandran, P. K., & Ozcan, P. (2017). A statistical analysis of the effects of Scrum and Kanban on software development projects. *Robotics and Computer-Integrated Manufacturing*, 43, 59-67.

## A Modified 3-Point Adams Block Method of the Variable Step Size Strategy for Solving Neutral Delay Differential Equations

Ubaidullah Yashkun<sup>1,2</sup>, Nurul Huda Abdul Aziz<sup>1</sup>

**Abstract:**

A modified 3-point Adams block method of order six (3ABM6) to solve neutral delay differential equations (NDDEs) using variable step size strategy is developed. The approximate solution of the retarded  $y(x-\tau)$  and the neutral terms  $y'(x-\tau)$  of the neutral delay differential equations at the grid points is obtained using the Newton divided difference interpolation technique. The proposed method will approximate the solution in each step using the three-points concurrently. To determine the performance of the proposed method, the maximum errors (MAXERR) and total number of function calls (FNC) will be compared with the method of 2-point order six predictor-corrector. The numerical results show that the 3ABM6 reduces the number of function calls and better accuracy in term of MAXERR.

**Keywords:** *Neutral Delay Differential Equations, Adams Block Method, Newton Interpolation Technique, Variable Step Size Technique.*

### 1. Introduction

In the fields of Science and Technology, the ordinary differential equations (ODEs) can be used to formulate the real-life problems by using initial value problem (IVP).

$$\frac{dy}{dx} = f(x, y) \quad a \leq x \leq b \quad (1)$$

Where, the function  $y$  usually represents physical quantities that evolve over time. Moreover, the delay differential equations (DDEs) are applied extensively in engineering, particularly the delay differential equations of the neutral type that usually arise in many scientific areas due to their ability to model many real-life phenomena. Neutral Delay Differential Equations do not depend only on the history of the function  $y(x-\tau)$  formulated but also on the history of the function derivative  $y'(x-\tau)$ .

$$\begin{aligned} y'(x) &= f(x, y(x), y(x-\tau(x)), y'(x), y'(x-\tau^*(x))) & x \geq x_0, \\ y(x) &= \phi(x) & \tau \leq x \leq x_0, \end{aligned} \quad (2)$$

Where,  $\tau(x) > 0$  and  $\tau^*(x) > 0$  are the delay arguments, and  $\phi(x)$  is the initial value function.

Several algorithms are envisaged in the literature to approximate the results of Neutral Delay Differential Equation (NDDEs) of type (2) which is  $y'(x)$  having  $f$  as an independent function. Neves [1], Bellen and Zennaro[2] and Al-Mutib[3] discussed algorithms based on one-step methods, while Jackiewicz and Lo[4] and Tavernini[5] discussed algorithms based on liner multistep methods. Fabiano and Payne[6] extended the spline approximation technique for Neutral Delay Differential Equations. The continuous Galerkin finite element method

<sup>1</sup> Institute of Engineering Mathematics, Universiti Malaysia Perlis, Kampus Pauh Putra, 02600, Arau, Perlis, Malaysia

<sup>2</sup> Sukkur IBA University, Airport Road, Sukkur, 65200, Sindh, Pakistan.

Corresponding Author: [ubaidullah@iba-suk.edu.pk](mailto:ubaidullah@iba-suk.edu.pk)

is used by Qin et al. [7] to solve the Linear Delay Differential Equations.

Wen et al. [8] discussed the numerical stability of NDDEs and Baker et al. [9] discussed the role of NDDEs in cell growth phenomena. The Neutral Functional Differential Equations (NFDEs) numerical solution is studied by Hu et al. [10] using linear multistep methods (LMM). The analytical and numerical stability of the nonlinear neutral delay-integral differential equations was studied by P. Hu and C. Huang [11]. An approximate solution for NFNDEs was also obtained by Wang and Li [12] using numerical methods, namely the one-leg  $\theta$  methods.

The Adams Block Methods (ABM) are efficient and computational cost-effective methods compared to the other numerical methods. Seong and Majid[13] studied direct two point order four and five multistep block method to solve the second order delay differential equations (DDEs). In their study the formulation and stability of the block method is discussed. Aziz et al. [14] presented the delay differential equations (DDEs) numerical solution using a predictor-corrector scheme in modified block method. ABM will compute the  $y$  approximate values in the block corresponding to the independent values of  $x$ . The one-block r-point technique for the 2nd order initial value problem is proposed by Fatunla [15]. A block method will compute simultaneously the solution values at numerous different points on the  $x$ -axis of the blocks by Ishak et al. [16] and Alkasassbeh and Omar [17].

In this paper, we introduce a new modified 3-point Adams block method of order six for first-order NDDEs. Some numerical results are shown in the tables for the proposed method. Here is the order of this paper. Section 2 formulation of the method is presented. In section 3, order, consistency and zero stability are discussed. In section 4 implementation of retarded and neutral terms are discussed. In section 5, the strategy of variable step size is discussed. Section 6, presents the algorithm of the method. Section 7, presents the numerical results, while discussions and conclusion are given in section 8.

## 2. The formulation of the 3-point Adams block method

Consider the first-order NDDEs (2) and discretize the interval  $[a, b]$  to subintervals

$$x_{n-4}, x_{n-3}, \dots, x_{n-4+k}, k = 0, 1, 2, \dots$$

such that  $a = x_{n-4}$  and  $b = x_{n-4+k}$ . From Fig. 1, the proposed method is used to

evaluate  $y_{n+1}, y_{n+2}$  and  $y_{n+3}$  corresponding to the grid values  $x_{n+1}, x_{n+2}$  and  $x_{n+3}$  having the variable step size  $h$  respectively. The four initial values  $y_{n-3}, y_{n-2}, y_{n-1}$  and  $y_n$  are obtained using the Euler method with the initial conditions corresponding to the values  $x_{n-3}, x_{n-2}, x_{n-1}$  and  $x_n$  having step ratios  $rh$  and  $qh$ .

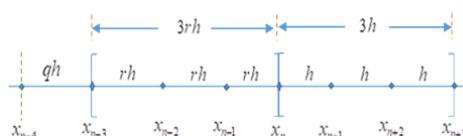


FIGURE 1: 3-Point Adams Block Method

Source: Author's own created

$y_{n+1}$  can be determine by integrating (1) as;

$$y(x_{n+1}) = y(x_n) + \int_{x_n}^{x_{n+1}} f(x, y) dx \tag{3}$$

In (3), to integrate the function  $f(x, y)$ , a Lagrange interpolation is substituted with the function. The approximation of  $y_{n+1}$  by using Maple with the ratio  $r$  is as follows:

$$y(x_{n+1}) = y(x_n) + \int_{x_n}^{x_{n+1}} P_4(x) dx \tag{4}$$

By substitution

$$s = \frac{x - x_n}{h} \text{ and } s = \frac{x - x_{n+3}}{h} \quad \text{3-point}$$

ABM of order six for the variable step size ratios is as follows:

For the Corrector when  $r = 1$  at the limit points  $-3$  to  $-2$ ,  $-2$  to  $-1$  and  $-1$  to  $0$

$$\begin{aligned}
 y_{n+1} = y_n &+ h \int_{-3}^{-2} \frac{1}{120} (s+5)(s+4)(s+3)(s+2)(s+1) ds f_{n+3} \\
 &- h \int_{-3}^{-2} \frac{1}{24} (s+5)(s+4)(s+3)(s+2)(s) ds f_{n+2} \\
 &+ h \int_{-3}^{-2} \frac{1}{12} (s+5)(s+4)(s+3)(s+1)(s) ds f_{n+1} \\
 &- h \int_{-3}^{-2} \frac{1}{12} (s+5)(s+4)(s+2)(s+1)(s) ds f_n \\
 &+ h \int_{-3}^{-2} \frac{1}{24} (s+5)(s+3)(s+2)(s+1)(s) ds f_{n-1} \\
 &- h \int_{-3}^{-2} \frac{1}{120} (s+4)(s+3)(s+2)(s+1)(s) ds * f_{n-2}
 \end{aligned}
 \tag{5}$$

$$\begin{aligned}
 y_{n+1} &= y_n + \frac{h}{1440} (11f_{n+3} - 93f_{n+2} + 802f_{n+1} + 802f_n - 93f_{n-1} + 11f_{n-2}) \\
 y_{n+2} &= y_{n+1} + \frac{h}{1440} (-27f_{n+3} + 637f_{n+2} + 1022f_{n+1} - 258f_n + 77f_{n-1} - 11f_{n-2}) \\
 y_{n+3} &= y_{n+2} + \frac{h}{1440} (475f_{n+3} + 1427f_{n+2} - 798f_{n+1} + 482f_n - 173f_{n-1} + 27f_{n-2})
 \end{aligned}
 \tag{6}$$

For the corrector when  $r = 2$  at the limit points  $-3$  to  $-2$ ,  $-2$  to  $-1$  and  $-1$  to  $0$

$$\begin{aligned}
 y_{n+1} &= y_n + \frac{h}{100800} (1328f_{n+3} - 9765f_{n+2} + 63952f_{n+1} + 46375f_n - 1183f_{n-1} + 93f_{n-2}) \\
 y_{n+2} &= y_{n+1} + \frac{h}{100800} (-2352f_{n+3} + 47285f_{n+2} + 65072f_{n+1} - 9975f_n + 847f_{n-1} - 77f_{n-2}) \\
 y_{n+3} &= y_{n+2} + \frac{h}{100800} (34288f_{n+3} + 93835f_{n+2} - 41328f_{n+1} + 15575f_n - 1743f_{n-1} + 173f_{n-2})
 \end{aligned}
 \tag{7}$$

For the corrector when  $r = \frac{1}{2}$  at the limit points  $-3$  to  $-2$ ,  $-2$  to  $-1$  and  $-1$  to  $0$

$$\begin{aligned}
 y_{n+1} &= y_n + \frac{h}{2520} (11f_{n+3} - 105f_{n+2} + 1211f_{n+1} + 1981f_n - 704f_{n-1} + 126f_{n-2}) \\
 y_{n+2} &= y_{n+1} + \frac{h}{2520} (-39f_{n+3} + 1057f_{n+2} + 1981f_{n+1} - 1029f_n + 704f_{n-1} - 154f_{n-2}) \\
 y_{n+3} &= y_{n+2} + \frac{h}{2520} (8011f_{n+3} + 2639f_{n+2} - 1869f_{n+1} + 2261f_n - 1728f_{n-1} + 406f_{n-2})
 \end{aligned}
 \tag{8}$$

The similar procedure can be used to get the one less order of predictor formulas.

### 3. Order Consistency and Zero Stability

In this study, the order of corrector is one order higher than the predictor. J. D. Lambert [18] gives the following definitions of order, consistency and zero-stability for the linear multistep methods. The order of this new developed 3ABM6 method is determined based on [18] and [13].

Theorem 1 (Lambert [18]). The general multistep method;

$$\sum_{m=0}^s \alpha_m y_{n+m} = h \sum_{m=0}^s \beta_m f(x_{n+m}, y_{n+m}) \quad (9)$$

Where,

$$\alpha_m = \begin{bmatrix} 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}, \quad \beta_m = \frac{1}{1440} \begin{bmatrix} 11 & -93 & 802 & 802 & -93 & 11 \\ -11 & 77 & -258 & 1022 & 637 & -27 \\ 27 & -173 & 482 & -798 & 1427 & 475 \end{bmatrix},$$

$$Y_{n+m} = [y_{n-2} \ y_{n-1} \ y_n \ y_{n+1} \ y_{n+2} \ y_{n+3}]^T$$

and

$$f_{n+m} = [f_{n-2} \ f_{n-1} \ f_n \ f_{n+1} \ f_{n+2} \ f_{n+3}]^T$$

From Theorem 1, the order of the corrector formulae of Equation (6) is

$$C_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C_1 = \alpha_1 + 2\alpha_2 + 3\alpha_3 + 4\alpha_4 + 5\alpha_5 + 6\alpha_6 - (\beta_0 + \beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6)$$

$$C_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + 2 \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} + 3 \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + 4 \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} + 5 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + 6 \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$- \frac{1}{1440} \left( \begin{bmatrix} 11 \\ -11 \\ 27 \end{bmatrix} + \begin{bmatrix} -93 \\ 77 \\ -173 \end{bmatrix} + \begin{bmatrix} 802 \\ -258 \\ 482 \end{bmatrix} + \begin{bmatrix} 802 \\ 1022 \\ -798 \end{bmatrix} + \begin{bmatrix} -93 \\ 637 \\ 1427 \end{bmatrix} + \begin{bmatrix} 11 \\ -27 \\ 475 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right)$$

$$C_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C_2 = \frac{1}{2!} \alpha_1 + \frac{2^2}{2!} \alpha_2 + \frac{3^2}{2!} \alpha_3 + \frac{4^2}{2!} \alpha_4 + \frac{5^2}{2!} \alpha_5 + \frac{6^2}{2!} \alpha_6 - (\beta_1 + 2\beta_2 + 3\beta_3 + 4\beta_4 + 5\beta_5 + 6\beta_6)$$

$$C_2 = \frac{1}{2!} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \frac{4}{2!} \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} + \frac{9}{2!} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + \frac{16}{2!} \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} + \frac{25}{2!} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \frac{36}{2!} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$- \frac{1}{1440} \left( \begin{bmatrix} -93 \\ 77 \\ -173 \end{bmatrix} + 2 \begin{bmatrix} 802 \\ -258 \\ 482 \end{bmatrix} + 3 \begin{bmatrix} 802 \\ 1022 \\ -798 \end{bmatrix} + 4 \begin{bmatrix} -93 \\ 637 \\ 1427 \end{bmatrix} + 5 \begin{bmatrix} 11 \\ -27 \\ 475 \end{bmatrix} + 6 \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right)$$

$$C_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C_3 = [0 \ 0 \ 0]^T, \quad C_4 = [0 \ 0 \ 0]^T, \quad C_5 = [0 \ 0 \ 0]^T, \quad C_6 = [0 \ 0 \ 0]^T$$

The order  $p \geq 1$  if and only if

$$C_0 = \sum_{m=0}^s \alpha_m = 0,$$

$$C_p = \sum_{m=0}^s \frac{m^k}{k!} \alpha_m - \sum_{m=0}^s \frac{m^{k-1}}{(k-1)!} \beta_m = 0, \quad k = 1, 2, \dots, p,$$

$$C_{p+1} = \sum_{m=0}^s \frac{m^{p+1}}{(p+1)!} \alpha_m - \sum_{m=0}^s \frac{m^p}{p!} \beta_m \neq 0. \quad (10)$$

The three-point block method can be written as matrix difference equation.

$$\alpha_m Y_{n+m} = h \beta_m f_{n+m}, \quad (11)$$

$$\begin{aligned}
 C_7 &= \frac{1}{7!}\alpha_1 + \frac{2^7}{7!}\alpha_2 + \frac{3^7}{7!}\alpha_3 + \frac{4^7}{7!}\alpha_4 + \frac{5^7}{7!}\alpha_5 + \frac{6^7}{7!}\alpha_6 \\
 &\quad - \left(\frac{1}{6!}\beta_1 + \frac{2^6}{6!}\beta_2 + \frac{3^6}{6!}\beta_3 + \frac{4^6}{6!}\beta_4 + \frac{5^6}{6!}\beta_5 + \frac{6^6}{6!}\beta_6\right) \\
 C_7 &= \frac{1}{7!} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \frac{2^7}{7!} \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} + \frac{3^7}{7!} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + \frac{4^7}{7!} \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} + \frac{5^7}{7!} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \frac{6^7}{7!} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \\
 &\quad - \frac{1}{1440} \left( \frac{1}{6!} \begin{bmatrix} -93 \\ 77 \\ -173 \end{bmatrix} + \frac{2^6}{6!} \begin{bmatrix} 802 \\ -258 \\ 482 \end{bmatrix} + \frac{3^6}{6!} \begin{bmatrix} 802 \\ 1022 \\ -798 \end{bmatrix} + \frac{4^6}{6!} \begin{bmatrix} -93 \\ 637 \\ 1427 \end{bmatrix} + \frac{5^6}{6!} \begin{bmatrix} 11 \\ -27 \\ 475 \end{bmatrix} + \frac{6^6}{6!} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right) \\
 C_7 &= \begin{bmatrix} -191 \\ 60480 \\ 271 \\ 60480 \\ -863 \\ 60480 \end{bmatrix} \neq 0.
 \end{aligned} \tag{12}$$

Equation (12) implies that  $C_7 \neq 0$  therefore, the order of the proposed method is order six.

**Definition 2.** The LMM in (9) is referred to as **consistent** such that order  $p \geq 1$ . The characteristics polynomials of LMM (9) are

$$\begin{aligned}
 \rho(\xi) &= \sum_{i=0}^k A_i^* \xi^i, \\
 \sigma(\xi) &= \sum_{i=0}^k B_i^* \xi^i.
 \end{aligned} \tag{13}$$

The LMM is consistent iff  $\rho(1) = 0$  and  $\rho'(1) = \sigma(1)$  (14)

**Definition 3.** Let the root  $|\xi| = 1$  for the polynomial  $\rho(\xi)$ , then the LMM (13) is referred to as **zero-stable**.

$$\begin{aligned}
 f[x_{n-4}, x_{n-3}, \dots, x_{n-4+k}] &= \frac{f[x_{n-3}, x_{n-2}, \dots, x_{n-4+k}] - f[x_{n-4}, x_{n-3}, \dots, x_{n-5+k}]}{x_{n-4+k} - x_{n-4}}, \\
 k &= 0, 1, 2, \dots
 \end{aligned} \tag{15}$$

### 5. The Strategy of Variable Step-Size

The variable step size strategy is recommended for this study. Step-size  $h$

### 4. Implementation of Retarded and Neutral Term

The first order NDDEs approximate solution depends on the retarded  $y(t - \tau)$  and neutral  $y'(t - \tau)$  terms. The delay of these terms may be constant  $(t - \tau)$ , time dependent  $(\tau(t))$  or state dependent  $(\tau(t, X(t)))$ . In this paper, the Newton divided difference technique is used to approximate these terms. For implementation, the required points are:

$$\begin{aligned}
 &(x_{n-4}, y_{n-4}), (x_{n-3}, y_{n-3}), (x_{n-2}, y_{n-2}), (x_{n-1}, y_{n-1}), \\
 &(x_n, y_n), (x_{n+1}, y_{n+1}), (x_{n+2}, y_{n+2}) \text{ and } (x_{n+3}, y_{n+3})
 \end{aligned}$$

The following recursion relation will be used for Newton divided differences.

control strategy will be like, the next step size is limited to double, as the previous step size when the computation proceeds as studied by [19]. The variable step size strategy may reduce the formulae that need

to be stored in the code, such that the storage capacity will increase. The evaluation technique is  $P(EC)^S$  or  $P(EC)^S E$ , such that  $P$  represents the predictor,  $E$  represents evaluation,  $C$  represents the corrector and  $S$  is the number of iterations. The  $P(EC)^S E$  mode is used if we need to calculate the function value of  $f_{n+i}^{(s)} = f(x_{n+i}, y_{n+i}^{(s)})$  otherwise we use  $P(EC)^S$  mode. Modified 3-point Adams block method of order six varies the step size  $h$ , to obtain three values during an integration step. Therefore, the efficiency of the method is acquired by taking the most optimal step size while reaching the desired accuracy.

### 6. Algorithm of the 3-point Adams Block Method of Order Six to Solve NDDEs

**Step 1:** Set TOL, initial condition  $x_0, y_0$  and step size  $h$

**Step 2:**  $y'(x) = f(x, y(x), y(x - \tau(x)), y'(x), y'(x - \tau(x)))$ .

**Step 3:**  $y(x) = \phi(x), y(0) = 1$ .

**Step 4:** For  $i = 1, 2, 3$  and  $4$  calculate  $x - \tau$  for  $y$  and its derivative. If  $x - \tau \leq x_0$ , go to 3 else calculate  $f(x, y)$  from 2, go to 5

**Step 5:**  $y_i = y_{i-1} + h * f_{i-1}$

**Step 6:** For  $i = 5, 6$  and  $7$  calculate  $x - \tau$  for  $y$  and its derivative. If  $x - \tau \leq x_0$ , go to 3 else go to 7

**Step 7:**

$$y(x_{n+1}) = y(x_n) + \frac{h}{1440} \sum_{i=1}^6 A_i f_{n-i+4}$$

**Step 8:** If  $|y_n^{(k+1)} - y_n^k| \leq 1.0 * TOL$  go to 9 else go to 4

**Step 9:** Stop

### 7. Problems and Numerical Results

In this section, three problems are present which demonstrates the implementation of the modified 3-point Adams block method of order six for first-order neutral delay differential equations (2). The computational results of the problems present confirm the efficiency and accuracy of the method. The analysis of the accuracy and efficiency of the method are based on maximum errors and the number of total steps taken. The approximate results are compared with the results of the 2-point order six predictor-corrector method. The maximum errors and number of total steps taken are plotted in Figs. 2-4.

**Problem 1. (Jackiewicz [20])**  
 $y'(x) = 0.75x^2y(x)/(1+x^2) + y(x/2) - (y'((x-1)/2))^2$ ,  
 $x \in [0,1], y(x) = 1 + x^2$ ,

$x \in [-0.5,0]$   
 The exact solution  
 $y(x) = 1 + x^2, x \in [-0.5,1]$

**Problem 2. (Jackiewicz [20])**  
 $y'(x) = \exp(1-2x^2)y(x^2)(y'(x-1/(1+x)))^{1+x}$ ,  $x \in [0,1]$ ,  
 $y(x) = \exp(x), x \in [-1,0]$ .

The exact solution is  
 $y(x) = \exp(x), x \in [-1,1]$ .

**Problem 3. (Jackiewicz [20])**  
 $y'(x) = 1 + y(x) - 2y^2(x/2) - y'(x - \pi)$ ,  $x \in [0, \pi]$ ,  
 $y(x) = \cos(x), x \in [-\pi, 0]$ .

The exact solution  
 $y(x) = \cos(x), x \in [-\pi, \pi]$ .

Following abbreviations are used in tables 1-3

- TOL:** Tolerance Defined
- MTD:** Employed Method
- TS:** Total Number of steps
- FS:** Number of failing steps

**MAXERR:** Maximum Error  
**FCN:** Number of function calls

**3ABM6:** Modified 3-Point Adams block method  
**N2PC6:** Neutral 2-point 6-order

**Table I:** Approximate results for Problem 1

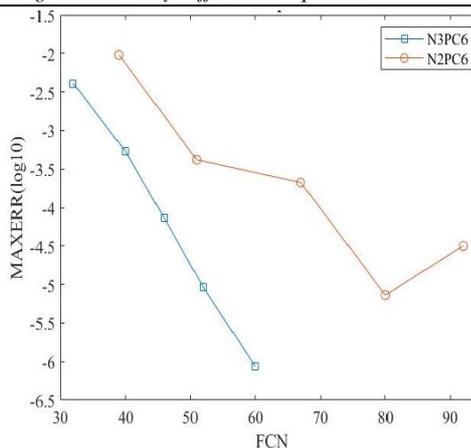
TOL	MTD	TS	FS	FNC	MAXERR	AVERR
$2^{-2}$	3ABM6	10	0	32	4.48E(-07)	1.86E(-07)
	N2PC6	11	0	39	1.42E(-05)	2.49E(-06)
$2^{-4}$	3ABM6	14	0	40	4.34E(-09)	2.84E(-09)
	N2PC6	14		51	1.24E(-08)	3.17E(-09)
$2^{-6}$	3ABM6	17	0	46	4.34E(-11)	2.75E(-11)
	N2PC6	18	0	67	1.71E(-09)	2.06E(-10)
$2^{-8}$	3ABM6	20	0	52	4.04E(-13)	2.68E(-13)
	N2PC6	21	0	80	3.23E(-13)	2.39E(-13)
$2^{-10}$	3ABM6	24	0	60	2.81E(-15)	2.44E(-15)
	N2PC6	24	0	92	1.90E(-11)	7.37E(-13)

**Table II:** Approximate results for Problem 2

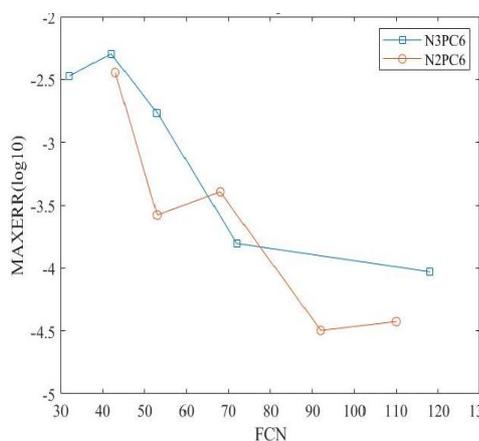
TOL	MTD	TS	FS	FNC	MAXERR	AVERR
$2^{-2}$	3ABM6	10	0	32	3.71E(-07)	1.50E(-07)
	N2PC6	11	0	43	3.94E(-07)	1.41E(-07)
$2^{-4}$	3ABM6	15	0	42	5.54E(-07)	7.03E(-08)
	N2PC6	14	0	53	2.14E(-09)	1.04E(-09)
$2^{-6}$	3ABM6	19	0	53	1.88E(-07)	2.34E(-08)
	N2PC6	19	0	68	3.27E(-09)	3.60E(-10)
$2^{-8}$	3ABM6	30	0	72	3.45E(-10)	7.45E(-10)
	N2PC6	27	0	92	1.91E(-11)	3.33E(-12)
$2^{-10}$	3ABM6	53	0	118	5.60E(-11)	1.25E(-11)
	N2PC6	33	0	110	6.11E(-12)	1.10E(-12)

**Table III:** Approximate results for Problem 3

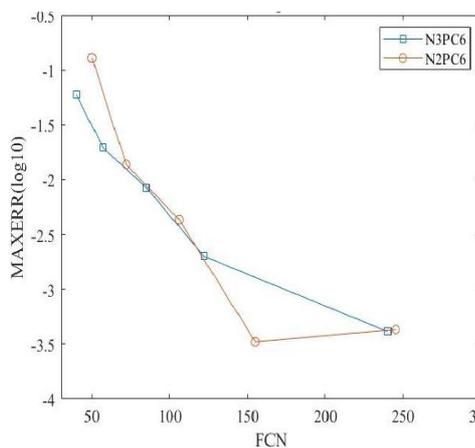
TOL	MTD	TS	FS	FNC	MAXERR	AVERR
$2^{-2}$	3ABM6	14	0	40	3.24E(-04)	2.40E(-05)
	N2PC6	13	0	50	7.03E(-04)	6.41E(-05)
$2^{-4}$	3ABM6	22	0	57	2.92E(-05)	1.76E(-06)
	N2PC6	17		72	2.04E(-05)	1.38E(-06)
$2^{-6}$	3ABM6	32	0	85	9.27E(-07)	1.54E(-07)
	N2PC6	25	0	106	4.73E(-07)	4.14E(-08)
$2^{-8}$	3ABM6	50	0	122	2.19E(-07)	1.96E(-08)
	N2PC6	41	0	155	2.69E(-09)	3.25E(-10)
$2^{-10}$	3ABM6	101	0	240	3.34E(-09)	3.74E(-10)
	N2PC6	62	0	245	9.46E(-10)	1.75E(-10)



**Fig 2:** Numerical Results Comparison of Problem 1.



**Fig 3:** Numerical Results Comparison of Problem 2.



**Fig 4:** Numerical Results Comparison of Problem 3.

## 8. Discussion and Conclusion

For problem 1, the total number of function calls and the maximum errors obtained by the modified 3-point Adams block method of order six are less than the 2-point order six PC method. For problem 2, the total number of function calls is less than the N2PC method, except for tolerance  $10^{-10}$  which is slightly high. Also, for problem 3, the total number of function calls is less than the N2PC method. However, for the given tolerances, the maximum errors for problems 1-3 are within the acceptable range, so both methods achieved the desired accuracy. It is clear from the approximate results that the proposed method is more efficient than the 2-point order six predictor-corrector method in solving NDDEs to the desired accuracy. Figures 2-4 represents the approximate solutions graphically of the problems 1-3. Our new proposed algorithm is efficient because during the iteration process the total number of function calls are less than the other algorithms used in literature.

In this paper, we have developed a modified 3-point Adams block method of order six based on the variable step size strategy for first-order neutral delay differential equations. The coding of the C programming language has been used to test the three numerical problems. The results obtained of the three problems tested indicate that the proposed method is more efficient and accurate in terms of the total number of function calls and maximum errors than the 2-point order six predictor-corrector method.

## REFERENCES

- [1] K. W. Neves, "Automatic integration of functional differential equations:," *an approach*, ACM Trans. Math. Softw. 1, pp. 357-368, 1975.
- [2] A. Bellen and M. Zennaro, "Numerical solution of delay differential equations by uniform corrections to an implicit Runge-

- Kutta method.” *Numer. Math.*, vol. 47, no. 2, pp. 301–316, 1985.
- [3] A. N. Al-Mutib, “Stability properties of numerical methods for solving delay differential equations,” *J. Comput. Appl. Math.*, vol. 10, no. 1, pp. 71–79, 1984.
- [4] Z. Jackiewicz and E. Lo, “The numerical solution of neutral functional differential equations by Adams predictor-corrector methods,” *Appl. Numer. Math.*, vol. 8, no. 6, pp. 477–491, 1991.
- [5] L. Tavernini, “Linear Multistep Methods for the Numerical Solution of Volterra Functional Differential Equations,” *Appl. Anal.*, vol. 3, no. 2, pp. 169–185, 1973.
- [6] R. H. Fabiano and C. Payne, “Spline approximation for systems of linear neutral delay-differential equations,” *Appl. Math. Comput.*, vol. 338, pp. 789–808, 2018.
- [7] H. Qin, Q. Zhang, and S. Wan, “The continuous Galerkin finite element methods for linear neutral delay differential equations,” *Appl. Math. Comput.*, vol. 346, no. 11501514, pp. 76–85, 2019.
- [8] L. Wen, X. Liu, and Y. Yu, “Stability of Runge-Kutta Methods for Neutral Delay Differential Equations,” *Discret. Dyn. Nat. Soc.*, vol. 2015, no. January 2015, 2015.
- [9] C. T. H. Baker, G. A. Bocharov, and F. A. Rihan, “Neutral Delay Differential Equations in the Modelling of Cell Growth,” *J. Egypt. Math. Soc.*, vol. 16, no. 2, pp. 133–160, 2008.
- [10] P. Hu, C. Huang, and S. Wu, “Asymptotic stability of linear multistep methods for nonlinear neutral delay differential equations,” *Appl. Math. Comput.*, vol. 211, no. 1, pp. 95–101, 2009.
- [11] P. Hu and C. Huang, “Analytical and numerical stability of nonlinear neutral delay integro-differential equations,” *J. Franklin Inst.*, vol. 348, no. 6, pp. 1082–1100, 2011.
- [12] W. S. Wang and S. F. Li, “On the one-leg  $\theta$ -methods for solving nonlinear neutral functional differential equations,” *Appl. Math. Comput.*, vol. 193, no. 1, pp. 285–301, 2007.
- [13] H. Y. Seong and Z. A. Majid, “Solving second order delay differential equations using direct two-point block method,” *Ain Shams Eng. J.*, vol. 8, no. 1, pp. 59–66, 2017.
- [14] N. H. A. Aziz and Z. A. Majid, “In Press , Accepted Manuscript – Note to users The Technique of Discontinuity Tracking Equations for Functional Differential Equations in 1-Point Implicit Method In Press , Accepted Manuscript – Note to users.”
- [15] S. O. Fatunla, “Block methods for second order odes,” *Int. J. Comput. Math.*, vol. 41, no. 1–2, pp. 55–63, 1991.
- [16] F. Ishak, Z. A. Majid, and M. B. Suleiman, “Efficient interpolators in implicit block method for solving delay differential equations,” *Int. J. Math. Comput. Simul.*, vol. 7, no. 2, pp. 116–124, 2013.
- [17] M. Alkasassbeh and Z. Omar, “Generalized Hybrid One-Step Block Method Involving Fifth Derivative for Solving Fourth-Order Ordinary Differential Equation Directly,” *J. Appl. Math.*, vol. 2017, 2017.
- [18] J. D. Lambert, “Computational Methods in Ordinary Differential Equations. London,” *London: Wiley*, 1973.
- [19] Z. Abdul Majid and M. Suleiman, “Predictor-corrector block iteration method for solving ordinary differential equations,” *Sains Malaysiana*, vol. 40, no. 6, pp. 659–664, 2011.
- [20] Z. Jackiewicz, “Numerische mathematik Adams methods for neutral functional,” *Differ. Equations*, vol. 230, pp. 221–230, 1982.

## Solution of Maintenance Strategy Selection Problem by using modified Fuzzy TOPSIS for of Material Handling Equipment

Alia Kausar<sup>1</sup>, Sana Akram<sup>1</sup>, Muhammad Farhan Tabassum<sup>2</sup>, Adeel Ahmad<sup>3</sup>,  
Shahzaman Khan<sup>4</sup>

---

### Abstract:

In today's competitive production environment, maintenance is one of the most important issues. Today in manufacturing plants productive methods are used to improve operating capacities that change environmental factors that lead to a competitive market. There is thus an important role to play in achieving those goals in selecting and explaining an optimal maintenance plan. Furthermore, there is a lack of an integrated model consisting of available requirements and choices, a systematic approach to maintenance instructions and strong maintenance decisions. In this paper modified Fuzzy TOPSIS method has been used for the solution of maintenance strategy selection problem. Linguistic variable and triangular fuzzy number have been used for modification in multi-criteria decision-making to solve maintenance strategy selection problem. Five experts have been considered for six types of maintenance strategy and ten decision criteria have been used in this problem. In this paper breakdown maintenance strategy best one out of all maintenance strategy for material handling equipment.

**Keywords:** *Maintenance strategy selection; Multi-criteria decision-making; Fuzzy TOPSIS method; Triangular fuzzy number; Linguistic variables.*

---

### 1. Introduction

Proper plant maintenance will significantly reduce total operating costs while increasing plant profitability. In order to provide maintenance personnel with improved technological and management skills [1], new technology and management experience must be emerging. In many companies, there is a strong motivation to optimize their equipment and plants ' life. This means that plants and

machinery can go further than their original design life. Damage and efficiency analysis has therefore recently become a critical tool to improve maintenance strategy, ensure safety and reduce costs [2].

Maintenance is an unavoidable cause of costs for many businesses. The maintenance function of these companies is corrective and is carried out only under emergency conditions. In view of certain key aspects such as product quality, protection of installations

---

<sup>1</sup> Department of Mathematics, Lahore Garrison University, Lahore, Pakistan.

<sup>2</sup> Department of Sports Sciences & Physical Education, Faculty of Allied Health, University of Lahore, Lahore, Pakistan.

<sup>3</sup> Department of Mathematics, Lahore Garrison University, Lahore, Pakistan.

<sup>4</sup> Department of Physical Education and Sports Sciences, Sukkur IBA University, Sukkur, Pakistan.

and rises in maintenance costs between 15% and 70% of total cost of production, this type of interference can now be no longer accepted. [3]. Many plants have different reliability, risk, and failure criteria for different machines. It is therefore evident that different maintenance strategies for different machines must be defined in a proper maintenance program. This makes it possible to preserve production plants' reliability and availability at an acceptable level and to prevent unnecessary investments to implement an improper maintenance strategy [4].

In the evaluation of related factors the problem of choosing maintenance approaches is a multi-criteria decision-making (MCDM). A number of solutions were proposed using fuzzy principles to solve this problem. The paper suggests a new maintenance strategy that can be the most effective maintenance strategies, depending on the level of uncertainty and also the nature and importance of the maintenance requirements [5].

Of fact, making the best possible alternative is the way to make choices. In some cases, decision-making problems are the main problem given other parameters. TOPSIS describes ideal solutions as positive and ideal solutions as negative. For the cost criteria and for the benefit criteria the positive solution is minimum and the negative ideal solution is minimum for benefit criteria and maximum for cost criteria [4]. In other types, the best possible values for the criteria consist of the optimal solution and all the worst values for the acquisition of the criteria are negative ideal solutions. As TOPSIS is an effective MCDM tool, a number of researchers have already used TOPSIS to solve many decision-making issues. Some of them de-fuzzy scores and weight in crisp values, other information has been lost during de-fuzzification. [6-8].

For the positive, ideal solution and the ideal negative solution based on criteria, Chen developed normalized values [9]. Standardized values are always  $(0, 0, 0)$  and  $(1, 1, 1)$  respectively for the ideal negative solution and

ideal positive solution on parameters.  $(0, 0, 0)$  and  $(1, 1, 1)$  are extraordinary values that could be a long way from true min and high values, so that TOPSIS' maximum and minimum estimates were not capable of extraordinary values. In Chen's work weight criteria are classified as triangular fuzzy numbers and triangular weight fuzzy numbers depending on loss of unusual values [5]. Chan's estimation is therefore very simple but incorrect triangle fuzzy numbers cannot be expressed. We have appointed fuzzy TOPSIS for the decision-making process of several criteria for a fluffy environment in order to avoid these problems [10].

The Fuzzy Technique for Order Preference by Similarity to Ideal Solution method (FTOPSIS) for the evaluation of maintenance strategies is used by some modification, In FTOPSIS Triangular Fuzzy Number (TFN) is used to model the uncertainty in the selection and the maintenance strategy selection problem is based on a fuzzy linguistic approach [11].

The remaining of this paper is organized as follows: in Section 2, the comprehensive detail of linguistic variable, fuzzy sets, and fuzzy numbers. In Section 3 the proposed fuzzy TOPSIS and the concepts behind it are introduced in details. In Section 4, the modified fuzzy TOPSIS method has been implemented for the solution of maintenance strategy selection problem. Finally, conclusions are given in Section 5.

## 2. Some Basic Concepts

Zadeh [12] first proposed the fuzzy set theory in order to deal with the vagueness of human thinking. A fuzzy collection is a class of things in which membership rates are constant. Such items are marked with a membership function that assigns a range of zero membership to one object. [12].

A linguistic variable is a variable with linguistic values. Intuitively easy-to-use linguistic terms were found to convey a

decision-maker's subjectivity and/or conceptual inaccuracy [13].

Depending on the situation, different fuzzy numbers can be used. It is often convenient for applications to work with triangle fuzzy number (TFN) and useful for promoting the representation and processing of information in a sophisticated environment because of its computational simplicity [14].

**Definition 1.** If all the decision makers are the highest in terms of performance of an alternative with respect to certain  $c_n$  parameters, then the alternative is the GPIS criteria  $c_n$ , named  $G^+$ .

**Definition 2.** If all the decision makers are the lowest in terms of performance of an alternative with respect to certain  $c_n$  parameters, then the alternative is the GPIS criteria  $c_n$ , named  $G^-$ .

In many MCDM implementations, such as supplier evaluation and selection, group decision-making and performance analysis, Fuzzy TOPSIS has been described as a dominant method [15]. Junior et al. (2014) conducted an analysis of the merits of the two techniques for MCDM problems between the fuzzy TOPSIS and the fuzzy analytical hierarchy (AHP) and found Fuzzy TOPSIS is the best way to find an ideal solution [16]. It can also be used to simplify the selection process and resolve ambiguities and uncertainty. Therefore, for a multi-criteria group decision-making scenario, we described Fuzzy TOPSIS process.

In this study TFNs are adopted in the fuzzy TOPSIS methods.

### 3. Proposed Fuzzy TOPSIS Method

Through *MAX* and *MIN* operations, we can find the positive ideal and negative ideal solutions, though against the criteria of positive ideal and negative ideal solution, this fuzzy number can be picked up by *MAX* and *MIN* operations which have been conceivable alternative cannot be found on rating. In this paper, another generally selected *TOPSIS* that

replaces  $U_p$  and  $L_o$  operations for *MAX* and *MIN* operations. By  $U_p$  and  $L_o$  operations, a set of fuzzy numbers is ranked quickly. On this occasion, we effectively find the ideal solution and negative ideal solution, and the fuzzy number can be found extra on these conceivable alternatives against the criteria of the positive ideal and negative ideal solution. The various steps of Fuzzy TOPSIS method are presented as follows:

**STEP 1:** Firstly, performance rating and weight are estimated with verbal terms. It represents the performances under linguistics classification, standardized by specialists, are: *very low (VL)*, *low (L)*, *medium low (ML)*, *medium (M)*, *medium high (MH)*, *high (H)* and *very high (VH)*.

Choosing committee of experts for decision-making. ( $E_k; k = 1, 2 \dots n$ ) and then to alternative  $M_i$  against choosing the criteria ( $C_j; j = 1, 2 \dots m$ ) where  $G_{ijk} = (g_{1ijk}, g_{2ijk}, g_{3ijk})$  is a triangular fuzzy number.

**STEP 2:** By using extension principle find the average performance rating of alternative  $M_i$  against criterion  $c_j$ , is  $G_{ij}$  as

$$G_{ijk} = (g_{1ijk}, g_{2ijk}, g_{3ijk}) \\ = 1/p \times (G_{ij1} + G_{ij2} + \dots + G_{ijp})$$

By the extension principle, we have

$$g_{1ij} = \sum_{k=1}^p \frac{g_{1ijk}}{p} \\ g_{2ij} = \sum_{k=1}^p \frac{g_{2ijk}}{p} \\ g_{3ij} = \sum_{k=1}^p \frac{g_{3ijk}}{p}$$

**STEP 3:** The performance ratings of alternative  $M_1, M_2, M_3 \dots$  composed a decision matrix that is,  $G = [G_{ij}]_{m \times n}$ ,  $[G_{i1}, G_{i2}, \dots, G_{in}]$  are the performance ratings of alternative  $M_i$ .

Let  $M^-$  and  $M^+$  are the negative ideal solution and positive ideal solution respectively;

Thus

$$M^- = [G_1^-, G_2^-, \dots, G_n^-] \text{ and}$$

$$M^+ = [G_1^+, G_2^+, \dots, G_n^+] \text{ and}$$

where  $G_j^- = L_o[G_{i1}, G_{i2}, \dots, G_{im}]$  and  $G_j^+ = U_p[G_{i1}, G_{i2}, \dots, G_{im}]$  for  $j = 1, 2, \dots, n$ .

**STEP 4:** Next find the distance from alternatives to the negative ideal solution (or positive ideal solution). Let  $d_{ij}^-$  and  $d_{ij}^+$  be the distance from  $G_{ij}$  to  $G_j^-$  and  $G_j^+$  respectively; where  $i = 1, 2, \dots, m; j = 1, 2, \dots, n$ . Let  $A = (a_1, a_2, a_3)$  and  $B = (b_1, b_2, b_3)$  be two triangular fuzzy numbers. So by the definition of distance,

$$d(A, B) = \sqrt{\frac{1}{3}[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2]}$$

**STEP 5:** Let  $W_{jk} = (w_{1jk}, w_{2jk}, w_{3jk})$  are weight evaluated by expert and  $E_k$  under criterion  $C_j$ , where  $j = 1, 2, \dots, n; k = 1, 2, \dots, p$

Suppose  $W_j$  be the average weight on criterion  $C_j$  is

$$W_j = (w_{1j}, w_{2j}, w_{3j}) = \frac{1}{p} \times (W_{j1} + W_{j2} + \dots + W_{jp})$$

where  $j = 1, 2, \dots, n$ .

From the extension principle,

$$w_{1j} = \sum_{k=1}^p \frac{w_{1jk}}{p}$$

$$w_{2j} = \sum_{k=1}^p \frac{w_{2jk}}{p}$$

$$w_{3j} = \sum_{k=1}^p \frac{w_{3jk}}{p}$$

**STEP 6:** The weight distance of alternative  $M_i$  to negative ideal solution  $M^-$  and ideal solution  $M^+$  respectively are find  $D_i^-$  and  $D_i^+$ .

$$D_i^- = \sum_{j=1}^n W_j \times d_{ij}^-$$

and

$$D_i^+ = \sum_{j=1}^n W_j \times d_{ij}^+,$$

where  $i = 1, 2, \dots, m$ .

**STEP 7:** Weighted distance of  $M_i$  can be find by  $[D_i^-, D_i^+]$ ;

$$LD^- = Lo(\{D_1^-, D_2^-, \dots, D_m^-\})$$

$$UD^- = Up(\{D_1^-, D_2^-, \dots, D_m^-\})$$

$$LD^+ = Lo(\{D_1^+, D_2^+, \dots, D_m^+\})$$

$$UD^+ = Up(\{D_1^+, D_2^+, \dots, D_m^+\})$$

**STEP 8:** From these two operations Lo and Up, the negative ideal solution is  $[LD^-, UD^+]$  and the ideal solution is  $[UD^-, LD^+]$  find for weighted distance of all alternatives.

**STEP 9:** Let  $M_i^-$  are the distance from  $[D_i^-, D_i^+]$  to  $[LD^-, UD^+]$ , and  $M_i^+$  denote the distance from  $[D_i^-, D_i^+]$  to  $[UD^-, LD^+]$ .

Define

$$M_i^- = d(D_i^-, LD^-) + d(D_i^+, UD^+)$$

And

$$M_i^+ = d(D_i^-, UD^-) + d(D_i^+, LD^+)$$

where  $i = 1, 2, \dots, m$ .

**STEP 10:** Closeness coefficient  $M_i^*$  of alternative  $A_i$  is defined as:

$$M_i^* = \frac{M_i^-}{M_i^- + M_i^+}$$

where  $i = 1, 2, \dots, m$ .

If  $M_i^* = 0$ , alternative  $M_i$  will be the poorest.

And  $M_i^* = 1$   $M_i$  is the best alternative.

#### 4. Maintenance Strategy Selection Problem for Material Handling Equipment

In this problem their five experts, such as:

$E_1 = \text{Expert 1}$

$E_2 = \text{Expert 2}$

$E_3 = \text{Expert 3}$

$E_4 = \text{Expert 4}$

$E_5 = \text{Expert 5}$

And six maintenance strategies:

$M_1 = \text{corrective maintenance,}$

$M_2 = \text{preventive maintenance,}$

$M_3 = \text{condition based maintenance}$

$M_4 = \text{opportunistic maintenance}$

$M_5 = \text{predictive maintenance}$

$M_6 = \text{breakdown Maintenance}$

Also this problem includes 10 evaluation criteria, such as:

- $C_1$  = quality
- $C_2$  = spare parts inventories
- $C_3$  = purchasing cost of spare parts
- $C_4$  = maintenance labour cost
- $C_5$  = reliability
- $C_6$  = safety
- $C_7$  = maintenance time
- $C_8$  = facilities
- $C_9$  = cost of supporting equipment
- $C_{10}$  = environment

The different weights of priority of each criterion and strategy are calculated using the fuzzy TOPSIS method following the

construction of the hierarchy. Linguistic variables are represented in Table 1.

**Table I. Fuzzy numbers and corresponding linguistic variables**

Linguistic Variables	Fuzzy Number
Very low (VL)	(0.0, 0.0, 0.1)
Low (L)	(0.0, 0.1, 0.3)
Medium low (ML)	(0.1, 0.3, 0.5)
Medium (M)	(0.3, 0.5, 0.7)
Medium high (MH)	(0.5, 0.7, 0.9)
High (H)	(0.7, 0.9, 1.0)
Very High (VH)	(0.9, 1.0, 1.0)

**Step 1:** The performance ratings of the six maintenance strategies in linguistic term is presented in Table 2.

**Table II. Experts have assigned the correct rating in terms of linguistic variables for each criterion**

$C_j$	$M_1$	$M_2$	$M_3$	$M_4$	$M_5$	$M_6$
$C_1$	H, VH, H, H, V	M, MH, M, M, L	H, H, VH, VH, M	M, MH, H, H, M	ML, M, MH, M	VH, H, VH, H, V
$C_2$	H, MH, M, MH	M, ML, M, ML, L	L, VL, L, ML, L	MH, H, H, MH	L, VL, ML, ML	VH, H, H, H, M
$C_3$	H, MH, M, MH	ML, M, M, ML	ML, M, MH, M	MH, H, M, MH	L, ML, M, ML	H, H, VH, VH, H
$C_4$	ML, M, H, H, M	H, M, L, ML, M	M, MH, M, ML	ML, ML, M, L	L, ML, ML, ML	H, VH, VH, H, V
$C_5$	H, VH, VH, H	ML, M, M, MH	H, H, VH, H, M	MH, H, M, M	ML, M, MH, M	H, H, VH, VH, H
$C_6$	H, MH, MH, H	MH, M, M, MH	H, H, VH, H, H	H, MH, M, M	M, MH, H, MH	H, H, VH, MH
$C_7$	H, MH, H, H, M	M, M, ML, ML	M, ML, ML, H, M	M, ML, MH, H	MH, M, ML, M	VH, H, VH, VH
$C_8$	H, VH, H, MH	MH, M, M, MH	VH, H, VH, H, V	M, ML, L, MH	H, MH, H, MH	MH, H, VH, VH
$C_9$	H, M, MH, M, L	M, ML, ML, H	M, MH, MH, ML	ML, ML, M, M	ML, L, ML, ML	VH, VH, H, VH
$C_{10}$	M, H, MH, H, V	H, MH, MH, H	VH, H, VH, VH	M, H, MH, M, M	H, H, H, MH, M	VH, H, H, VH, H

**Step 2:** The average ratings of the six maintenance strategies is in Table 3.

**Table III. Average ratings of maintenance strategies**

$C_j$	$M_1$	$M_2$	$M_3$	$M_4$	$M_5$	$M_6$
$C_1$	0.78,0.94,1.0	0.38,0.58,0.7	0.74,0.90,0.9	0.54,0.74,0.9	0.34,0.54,0.7	0.82,0.96,1.0
$C_2$	0.54,0.74,0.9	0.16,0.34,0.5	0.02,0.12,0.3	0.54,0.74,0.9	0.02,0.16,0.3	0.70,0.88,0.9
$C_3$	0.54,0.74,0.9	0.30,0.50,0.6	0.26,0.46,0.6	0.50,0.70,0.8	0.16,0.34,0.5	0.78,0.94,1.0
$C_4$	0.42,0.62,0.7	0.24,0.42,0.6	0.26,0.46,0.6	0.16,0.34,0.5	0.12,0.30,0.5	0.82,0.96,1.0
$C_5$	0.74,0.90,0.9	0.34,0.54,0.7	0.70,0.88,0.9	0.50,0.70,0.8	0.26,0.46,0.6	0.78,0.94,1.0
$C_6$	0.58,0.78,0.9	0.46,0.66,0.8	0.74,0.92,1.0	0.46,0.66,0.8	0.46,0.66,0.8	0.66,0.84,0.9
$C_7$	0.62,0.82,0.9	0.30,0.50,0.6	0.26,0.46,0.6	0.42,0.62,0.8	0.20,0.38,0.5	0.86,0.98,1.0
$C_8$	0.66,0.84,0.9	0.38,0.58,0.8	0.82,0.96,1.0	0.20,0.38,0.5	0.58,0.78,0.9	0.78,0.92,0.9
$C_9$	0.46,0.66,0.8	0.34,0.54,0.7	0.38,0.58,0.7	0.18,0.38,0.5	0.12,0.30,0.5	0.86,0.98,1.0

$C_{10}$	0.62,0.80,0.9	0.62,0.82,0.9	0.82,0.96,1.0	0.46,0.66,0.8	0.62,0.82,0.9	0.78,0.91,1.0
----------	---------------	---------------	---------------	---------------	---------------	---------------

**Step 3:** By the performance ratings of alternative composed a decision-making matrix  $G$  is in Table 4.

**Table IV. Decision-making matrix**

$G_1^+ = 0.82, 0.96, 1.00$	$G_6^+ = 0.74, 0.92, 1.00$	$G_7^- = 0.34, 0.54, 0.74$	$G_6^- = 0.46, 0.66, 0.84$
$G_2^+ = 0.70, 0.88, 0.98$	$G_7^+ = 0.86, 0.98, 1.00$	$G_2^- = 0.02, 0.12, 0.30$	$G_7^- = 0.20, 0.38, 0.58$
$G_3^+ = 0.78, 0.94, 1.00$	$G_8^+ = 0.82, 0.96, 1.00$	$G_3^- = 0.16, 0.34, 0.54$	$G_8^- = 0.20, 0.38, 0.58$
$G_4^+ = 0.82, 0.96, 1.00$	$G_9^+ = 0.86, 0.98, 1.00$	$G_4^- = 0.12, 0.30, 0.50$	$G_9^- = 0.12, 0.30, 0.50$
$G_5^+ = 0.78, 0.94, 1.00$	$G_{10}^+ = 0.82, 0.96, 1.00$	$G_5^- = 0.26, 0.46, 0.66$	$G_{10}^- = 0.46, 0.66, 0.84$

**Step 4:** The distance values for the six maintenance strategies on 10 criteria is in Table 5.

**Table V. Distance values for maintenance strategies**

$C_j$	$M_1$		$M_2$		$M_3$	
	$d(G_{1j}, G_{j+})$	$d(G_{1j}, G_{j-})$	$d(G_{2j}, G_{j+})$	$d(G_{2j}, G_{j-})$	$d(G_{3j}, G_{j+})$	$d(G_{3j}, G_{j-})$
$C_1$	0.08246	0.3747	0.3589	0.0400	0.0589	0.3402
$C_2$	0.1311	0.5816	0.5089	0.2046	0.7077	0.0000
$C_3$	0.1894	0.3804	0.4189	0.1469	0.4533	0.1137
$C_4$	0.3286	0.3004	0.5125	0.1137	0.4758	0.1536
$C_5$	0.0346	0.4189	0.3747	0.0800	0.0589	0.3968
$C_6$	0.1275	0.1137	0.2392	0.0000	0.0000	0.2392
$C_7$	0.1681	0.4141	0.4642	0.1071	0.5033	0.0673
$C_8$	0.1178	0.4350	0.3514	0.2082	0.0000	0.5469
$C_9$	0.3098	0.3468	0.4252	0.2269	0.03824	0.2735
$C_{10}$	0.1549	0.1311	0.1428	0.1479	0.0000	0.2859
$C_j$	$M_6$		$M_6$		$M_6$	
	$d(G_{4j}, G_{j+})$	$d(G_{4j}, G_{j-})$	$d(G_{5j}, G_{j+})$	$d(G_{5j}, G_{j-})$	$d(G_{6j}, G_{j+})$	$d(G_{6j}, G_{j-})$
$C_1$	0.2135	0.1876	0.3977	0.0000	0.0000	0.3977
$C_2$	0.1311	0.5816	0.6808	0.0327	0.0000	0.7077
$C_3$	0.2239	0.3468	0.5645	0.0000	0.0000	0.5540
$C_4$	0.5864	0.0400	0.6260	0.0000	0.0000	0.6260
$C_5$	0.2277	0.2274	0.4533	0.0000	0.0000	0.4532
$C_6$	0.2392	0.0000	0.2392	0.0000	0.0693	0.1701
$C_7$	0.3479	0.2269	0.5692	0.0000	0.0000	0.5692
$C_8$	0.5469	0.0000	0.1736	0.3934	0.0346	0.5125
$C_9$	0.5770	0.7394	0.6481	0.0000	0.0000	0.6481
$C_{10}$	0.2859	0.0000	0.1428	0.1479	0.0258	0.2622

**Step 5:** The linguistic weights and average ratings for 10 criteria are in Table 6-7.

**Table VI. Linguistic weights for criteria**

$C_j$	$E_1$	$E_2$	$E_3$	$E_4$	$E_5$
$C_1$	VH	H	VH	VH	H
$C_2$	MH	M	ML	L	VL
$C_3$	M	ML	M	MH	ML
$C_4$	MH	H	ML	M	M
$C_5$	VH	H	H	VH	VH
$C_6$	H	VH	H	VH	VH
$C_7$	MH	M	H	MH	H
$C_8$	H	MH	VH	H	MH
$C_9$	L	ML	M	M	VL
$C_{10}$	MH	H	VH	H	MH

**Table VII. Average ratings**

$C_j$	$E_1$	$E_2$	$E_3$	$E_4$	$E_5$
$C_1$	0.9,1.0,1.0	0.7,0.9,1.0	0.9,1.0,1.0	0.9,1.0,1.0	0.7,0.9,1.0
$C_2$	0.5,0.7,0.9	0.3,0.5,0.7	0.1,0.3,0.5	0.0,0.1,0.3	0.0,0.0,0.1
$C_3$	0.3,0.5,0.7	0.1,0.3,0.5	0.3,0.5,0.7	0.5,0.7,0.9	0.1,0.3,0.5
$C_4$	0.5,0.7,0.9	0.7,0.9,1.0	0.1,0.3,0.5	0.3,0.5,0.7	0.3,0.5,0.7
$C_5$	0.9,1.0,1.0	0.7,0.9,1.0	0.7,0.9,1.0	0.9,1.0,1.0	0.9,1.0,1.0
$C_6$	0.7,0.9,1.0	0.9,1.0,1.0	0.7,0.9,1.0	0.9,1.0,1.0	0.9,1.0,1.0
$C_7$	0.5,0.7,0.9	0.3,0.5,0.7	0.7,0.9,1.0	0.5,0.7,0.9	0.7,0.9,1.0
$C_8$	0.7,0.9,1.0	0.5,0.7,0.9	0.9,1.0,1.0	0.7,0.9,1.0	0.5,0.7,0.9
$C_9$	0.0,0.1,0.3	0.1,0.3,0.5	0.3,0.5,0.7	0.3,0.5,0.7	0.0,0.0,0.1
$C_{10}$	0.5,0.7,0.9	0.7,0.9,1.0	0.9,1.0,1.0	0.7,0.9,1.0	0.5,0.7,0.9

Calculated average weights against the 10 criteria from Table 7 are:

$$\begin{aligned}
 W_1 &= (0.82,0.96,1.00) \\
 W_2 &= (0.18,0.32,0.50) \\
 W_3 &= (0.26,0.46,0.66) \\
 W_4 &= (0.38,0.58,0.76) \\
 W_5 &= (0.82,0.96,1.00) \\
 W_6 &= (0.82,0.96,1.00) \\
 W_7 &= (0.54,0.74,0.90) \\
 W_8 &= (0.66,0.84,0.96) \\
 W_9 &= (0.14,0.28,0.46) \\
 W_{10} &= (0.66,0.84,0.96)
 \end{aligned}$$

**Step 6:** The weighted distance values of six maintenance strategies on 10 criteria are presented as follows.

$$\begin{aligned}
 D_1^+ &= (1.3209,1.7071,1.9826) \\
 D_2^+ &= (1.8293,2.4644,2.9810) \\
 D_3^+ &= (0.8680,1.3035,1.7613) \\
 D_4^+ &= (1.6803,2.2561,2.7167) \\
 D_5^+ &= (2.0081,2.7557,3.3931) \\
 D_6^+ &= (0.0969,0.1175,0.1276) \\
 D_1^- &= (1.7075,2.2854,2.7531) \\
 D_2^- &= (0.5413,0.7561,0.9483) \\
 D_3^- &= (1.5127,1.9045,2.1538) \\
 D_4^- &= (0.7764,1.1422,1.5094) \\
 D_5^- &= (0.4161,0.5593,0.6831)
 \end{aligned}$$

$$D_6^- = (2.2559, 3.0780, 3.7704)$$

**Step 7:** The values of weighted distance are given below

$$LD^+ = (0.0969, 0.1175, 0.1276)$$

$$UD^+ = (2.0081, 2.7557, 3.3931)$$

$$LD^- = (0.4161, 0.5593, 0.6831)$$

$$UD^- = (2.2559, 3.0780, 3.7704)$$

**Step 8:** Positive and negative ideal solutions from weighted distance are given below

$$d(D_1^+, UD^+) = 1.0893$$

$$d(D_1^+, LD^+) = 1.5774$$

$$d(D_2^+, UD^+) = 0.8378$$

$$d(D_2^+, LD^+) = 2.3559$$

$$d(D_3^+, UD^+) = 1.4224$$

$$d(D_3^+, LD^+) = 1.2477$$

$$d(D_4^+, UD^+) = 0.5209$$

$$d(D_4^+, LD^+) = 2.1435$$

$$d(D_5^+, UD^+) = 0.0000$$

$$d(D_5^+, LD^+) = 2.6629$$

$$d(D_6^+, UD^+) = 2.6629$$

$$d(D_6^+, LD^+) = 0.0000$$

and

$$d(D_1^-, UD^-) = 0.8091$$

$$d(D_1^-, LD^-) = 1.7255$$

$$d(D_2^-, UD^-) = 2.3532$$

$$d(D_2^-, LD^-) = 0.2039$$

$$d(D_3^-, UD^-) = 1.2306$$

$$d(D_3^-, LD^-) = 1.3134$$

$$d(D_4^-, UD^-) = 1.9191$$

$$d(D_4^-, LD^-) = 0.6198$$

$$d(D_5^-, UD^-) = 2.5338$$

$$d(D_5^-, LD^-) = 0.0000$$

$$d(D_6^-, UD^-) = 0.0000$$

$$d(D_6^-, LD^-) = 2.5338$$

**Step 9:** From these above distance values,  $M_i^+$  and  $M_i^-$  can be calculated:

$$M_1^+ = 2.3865 \quad M_1^- = 2.8148$$

$$M_2^+ = 4.7091 \quad M_2^- = 1.0417$$

$$M_3^+ = 2.4783 \quad M_3^- = 2.7358$$

$$M_4^+ = 4.0626 \quad M_4^- = 1.1407$$

$$M_5^+ = 5.1967 \quad M_5^- = 0.0000$$

$$M_6^+ = 0.0000 \quad M_6^- = 5.1967$$

**Step 10:** Finally, all the results are evaluated, final Scores and ranks are given in Table 8.

$$M_1^* = 0.5412$$

$$M_2^* = 0.1811$$

$$M_3^* = 0.5247$$

$$M_4^* = 0.2192$$

$$M_5^* = 0.0000$$

$$M_6^* = 1.0000$$

**Table VIII. Final Scores and ranks of strategies**

Strategy	Final Scores	Ranks
$M_1$	0.5412	2
$M_2$	0.1811	5
$M_3$	0.5247	3
$M_4$	0.2192	4
$M_5$	0.0000	6
$M_6$	1.0000	1

Clearly, from Table 8 the ranking order is  $M_6 > M_1 > M_3 > M_4 > M_2 > M_5$ . Therefore, performances of  $M_6$  is the best.

## 5. Result and Discussion

The result reported in [17] the same problem has been solved with the help of fuzzy SAW method the order ranking of maintenance strategy for material handling equipment. The results are as follows

$$M_6 > M_1 > M_3 > M_4 > M_2 > M_5.$$

And from the above experimentation we have the same results which showed that breakdown maintenance  $M_6$  is the best maintenance strategy for material handling equipment and predictive maintenance  $M_5$  is the poor maintenance strategy for material handling equipment.

## 6. Conclusion

In this paper modified Fuzzy TOPSIS method has been used for the solution of maintenance strategy selection problem. Linguistic variable and triangular fuzzy number have been used for modification in multi-criteria decision-making to solve maintenance strategy selection

problem. Five experts have been considered for six types of maintenance strategy and ten decision criteria have been used in this problem. Two operators  $U_p$  and  $L_o$ , which satisfied fuzzy numbers of partial ordering relations for generalized of TOPSIS, these two operations are used to determine the negative ideal and positive ideal solutions in a fuzzy environment. Ultimately, it is concluded that the malfunction management approach for equipment for material handling is best of all maintenance strategies.

#### REFERENCES

- [1] Bertolini, M. and M. Bevilacqua, *A combined goal programming—AHP approach to maintenance selection problem*. Reliability Engineering & System Safety, 2006. **91**(7): p. 839-848.
- [2] Yatomi, M., et al., *Application of risk-based maintenance on materials handling systems*. IHI engineering review, 2004. **37**(2): p. 52-58.
- [3] Sarkar, A., D.K. Behera, and B. Sarkar, *The maintenance strategy selection of a gas turbine power plant system*. Journal of Information and Operations Management, 2011. **2**(1): p. 9.
- [4] Akhshabi, M., *A new fuzzy multi criteria model for maintenance policy*. Middle-East Journal of Scientific Research, 2011. **10**(1): p. 33-38.
- [5] Saeed, M. and M. Tabassum, *COMPARISON BETWEEN FUZZY SOFT MATRIX (FSM) AND INTERVAL VALUED FUZZY SOFT MATRIX (IVFSM) IN DECISION MAKING*. Science International, 2016. **28**(5).
- [6] Saeed, M., et al., *Generalization of TOPSIS from Soft Set to Fuzzy Soft Sets in Decision Making Problem*. Scientific Inquiry Review, 2017. **1**(1): p. 11-18.
- [7] Tsaur, S.-H., T.-Y. Chang, and C.-H. Yen, *The evaluation of airline service quality by fuzzy MCDM*. Tourism management, 2002. **23**(2): p. 107-115.
- [8] Ahmad, A., et al., *Multi-Criteria Decision-Making for Airport Operation Performance Using Triangular Fuzzy Numbers*. Scientific Inquiry Review, 2019. **3**(3): p. 01-15.
- [9] Chen, C.-T., *Extensions of the TOPSIS for group decision-making under fuzzy environment*. Fuzzy sets and systems, 2000. **114**(1): p. 1-9.
- [10] Gani, A.N. and S.M. Assarudeen, *A new operation on triangular fuzzy number for solving fuzzy linear programming problem*. Applied Mathematical Sciences, 2012. **6**(11): p. 525-532.
- [11] Omar, M.N. and A.R. Fayek, *A TOPSIS-based approach for prioritized aggregation in multi-criteria decision-making problems*. Journal of Multi-Criteria Decision Analysis, 2016. **23**(5-6): p. 197-209.
- [12] Zadeh, L.A., *Fuzzy sets*. Information and control, 1965. **8**(3): p. 338-353.
- [13] Zadeh, L.A., *The concept of a linguistic variable and its application to approximate reasoning-III*. Information sciences, 1975. **9**(1): p. 43-80.
- [14] Ertugrul, İ. and N. Karakaşoğlu, *Comparison of fuzzy AHP and fuzzy TOPSIS methods for facility location selection*. The International Journal of Advanced Manufacturing Technology, 2008. **39**(7-8): p. 783-795.
- [15] Bottani, E. and A. Rizzi, *A fuzzy TOPSIS methodology to support outsourcing of logistics services*. Supply Chain Management: An International Journal, 2006. **11**(4): p. 294-308.
- [16] Junior, F.R.L., L. Osiro, and L.C.R. Carpinetti, *A comparison between Fuzzy AHP and Fuzzy TOPSIS methods to supplier selection*. Applied Soft Computing, 2014. **21**: p. 194-209.
- [17] Sagara, M.K., P. Jayaswala, and K. Kushwahc, *Exploring Fuzzy SAW Method for Maintenance Strategy Selection Problem of Material Handling Equipment*. 2013.

## A Review of Virtual Touch Interaction Systems

Rizwan Ali Abro<sup>1</sup>, Ahsanullah Abro<sup>1</sup>, Sana Fatima<sup>1</sup>, Sarang Shaikh<sup>1</sup>, Sindhu Abro<sup>1</sup>

---

### Abstract:

Virtual touch has opened a new dimension in the field of Human Computer Interaction (HCI). Enabled with the motion and gesture detection techniques, it allows manual interaction with virtual environments. Virtual environments are built for performing tasks which are typically achieved using hands in the real world, such as handling objects. An analysis on wearables, graspable and touchable haptic devices used in multiple fields is discussed in this study. This paper reviews the design aspect as well as the usage of the haptic devices along with trendy practices that involved in generating satisfactory touch sensations.

**Keywords:** *Virtual Touch, Human Computer Interaction, Review, touch sensations*

---

### 1. Introduction

The sense of touch-Haptics empowers humans to explore and manipulate the tasks with wide variety in real world. The sense of touch must provide the realism to the virtual worlds and robot teleoperation by artificially recreating by stimulated human body (specially the hands). This paper provides the state of the art of design, control and the applications of artificial human sensations that are generated by the haptic devices. There are many areas which can be covered through the haptic technology but the most important to mention in this article is robotic haptics (in this haptic technology the robots enable to get the sense of touch using the force and tactile sensors by using algorithm [1], and invasive sense of touch stimulating haptic (enabling sense of touch-haptic in humans and animals by stimulating peripheral nervous system by electrically [2].

Haptic plays a crucial role and it is quite arduous to expect life without haptics, which

is elemental and innate part of life. Sensing and manipulating the real-world objects without haptics imagining the life without water, and can be cause of failure in determining the texture and surface properties [6]. As technology is sphering and growing its impact in all aspects of life as well as business world, the latest technological elements are therefore much needed in every field, for instance, specialized professions can adopt the artificial sense of touch and haptic feedback for remote tasks, astronauts can teleoperate the robots in the space for repairing the space machines, in medical field the surgeons can use the robots in sensitive areas where human hands enable to reach, online customers can sense the stuff of the online products with artificial sense of touch. The human operators are aimed to accord the sense of touch in such teleoperation scenarios, in order to make them feel that they are manipulating the objects and environment like

---

<sup>1</sup> Department of Computer Science, Sukkur IBA University Sukkur, Sindh, Pakistan  
Corresponding Author: [rizwan.ali@iba-suk.edu.pk](mailto:rizwan.ali@iba-suk.edu.pk)

they interact with the real-world objects with their own hands.

In a few cases, the patients who lost their touch sensation in any disease or accident injury this study has suggested for replacing the sense of touch [11] [12] [13]. Many people lose the sense of touch if an upper-limb amputee is lost because of loss of a hand [14]. In such cases the artificial prosthetic hand used to sense the haptic feedback by interacting with the objects by getting back interaction information from the amputee, as amputee must not rely on the sight itself for manipulating the objects.

The sense of touch is not bounded to a certain area of the human body, contrary to rest four senses (sight, hearing, taste, and smell) [15]. The sense of touch can be found in entire human body particularly tendons, muscles, joints, skin and sensory organs. Usually the sense of touch is split into two modalities: kinesthetic and tactile. The kinesthetic is sensed in the joints, muscles and the tendons, it is usually denominated as torques or forces. While the tactile are purely sensed by the special sensory organs denominated as mechanoreceptors which are present in the skin. It responds to a particular kind of haptic stimulus after sensing from each kind of mechanoreceptor [16] [17] [18] [19] [20].



**Fig 1:** Visual demonstration of Haptic Devices

The mechanoreceptor is restricted and represented to its temporal determination and stick with its susceptible fields [20]. There are two variations in the mechanoreceptors, the fast-adapting mechanoreceptors and the slow-adapting mechanoreceptors. The fast-adapting

mechanoreceptors is responsible to capture temporary signals while the slow-adapting mechanoreceptors usually capture the static stimuli. For instance, the Meissner corpuscles are example of fast-adapting mechanoreceptors which react with the lower frequency vibrations and be able to sense the skin rate deformation [3].

## 2. Literature Review

The major step to perform the literature review was to collect the recent and the relevant articles from the reliable and identified sources as research questions can be answered. There are three types of haptic devices Graspable, Wearable and touchable [8]. The focus of this review is on all the three categories. furthermore, the presented study is a systematic literature review is on these categories. This study reviews the technology behind the haptic devices in three categories to answer following research questions:

1. What are the most effective and known performance parameters to evaluate the characteristics of haptic devices?
2. What are the most influential devices available in three types of haptic devices (Graspable, Wearable and Touchable)?

The sources which used for the searching the literature were identified by searching the IEEE digital library, ACM library, Springer Link Library and Google Scholar. The searching keywords were “Sense of touch” “Haptic Touch” and “Virtual Touch”. The number of conferences and journals were identified are: (IEEE Robotics and Automation Letter) IEEE RAL, (Benelux Conference on Artificial Intelligence) BNAIC, (ACM on Human-Computer Interaction) ACM HCI, (Virtual, Augmented and Mixed Reality) VAMR, (International Journal of Advanced Robotic Systems) IJARS, (International Conference on Ubiquitous Computing and Ambient Intelligence, Ubiquitous Computing and Ambient Intelligence) UCAmI.

**Table 1:** Number of Publications by Source

Source	2018	2017	2016	2015	2014	2013	Total
<b>IEEE RAL</b>	117	188	96	114	132	103	<b>750</b>
<b>BNAIC</b>	33	29	93	55	81	45	<b>336</b>
<b>ACM HCI</b>	179	155	99	125	88	110	<b>756</b>
<b>VAMR</b>	65	45	70	54	82	89	<b>405</b>
<b>IJARS</b>	66	96	145	66	49	111	<b>533</b>
<b>UCAmI</b>	37	52	61	44	41	54	<b>289</b>

**Table 2:** Number of relevant publications by source

Source	2018	2017	2016	2015	2014	2013	Total
<b>IEEE RAL</b>	21	16	31	25	9	15	<b>117</b>
<b>BNAIC</b>	5	4	18	23	14	13	<b>77</b>
<b>ACM HCI</b>	8	12	16	9	5	6	<b>56</b>
<b>VAMR</b>	11	9	15	14	15	8	<b>72</b>
<b>IJARS</b>	4	8	5	11	19	3	<b>50</b>
<b>UCAmI</b>	6	15	14	18	20	16	<b>89</b>

The selection of the papers is based on the most relevant information available in the research paper which addresses the usage of the haptic devices based on said categories. The sources were extracted based on the keywords selection which matched this papers requirement. There were 70 papers included in this study but later the number was decreased because this study has to address and answer the research questions. The literature review was narrowed based on the publications between the years 2013 to 2018 because of need of the haptic devices in the said period. Table 1 highlighted the publications numbers which have been examined from each source. The sources used in this study has different numbers of publications, such as, (Journal Articles, short

papers, Full papers). this study focuses on the conference and journal articles.

The haptic word is derived from the Greek word “haptesthai” which is described as the ability to manipulate or touch the objects [55]. There are two parts of the haptic systems which are human and machine parts. The human part is responsible for sensing and controlling the position of hand whereas machine part helps to simulate the contact with a virtual object by exerting forces from the hand [56].

extensive use of the wearable devices which make use of the haptic rendering techniques that allows users to manipulate variety of objects. For example, a simple haptics aided glove paired with wrist sensors can assist to have the sense of wrist orientation. With the help of virtual reality patients are made to

perform the motion related tasks during training sessions to reach the goals of increasing complexity [57]. Haptic technology was introduced by military field specially for astronauts and pilots' training simulation. As the advancements in computer technologies took place gaming industry began to use this technology in video games such as Oculus Rift, which is an example of wearable device [58].

Increasing usage of graspable devices has been witnessed in medical training sessions as well as in surgeries as the use of VR Robotic technology has enhanced [59].

Haptic system in robotic technology is considered to be a channel that can provide force sensations from real environment to the users. This system is referred to the handling and application of touch sensation to interact with machines. Such as haptic telepresence robots or other touchable devices are considered in this category [60].

In order to provide sensory feedback, haptic applications are equipped with specialized hardware. Commonly used haptic interface configuration uses the mechanical linkages to connect a person's finger with computer interface. Whenever user moves its finger the translated actions are highlighted on the screen [61].

### 3. Results and Discussions

The paper discussed the various aspects in haptic devices which are highlighted and listed first in the literature. Many haptic devices are available in all three categories. The usage of the haptic devices for various purposes which includes the most commonly defined property such as availability of the devices, which devices has most impact and which device is preferred by the user based on reliability and price. It is obvious that haptic device ought to fascinate the user requirements by considering the reliability and prices factors. Many researchers have addressed this issue that the device functionality is directly proportional to its price [14].

There is a significant increase in the use of technology in almost every field so it is required to switch the number of aesthetically potential applications to haptics. The use of haptics is witnessed in communication, online stores, medical and gaming [35]. Throughout the study it has seen that impact of the haptics in different areas where the human interaction is almost impossible or hard to tackle the situation, the sense of touch is welcoming to play its role in those situations, for instance, robotic distance surgery, prosthetic organs) that touch has ability to replace the visual or auditory senses cues and touch can augment the virtual interactions. One of the major limiting reasons in the field of haptics is lack of availability of hardware and its expressiveness. Limited number of actuators available commercially for designing haptic output exclusively and it bring many researchers to make the haptic devices at their own using off the shelf components [38,42]. Focusing on the perceptual abilities of human sense of touch during the process of design is a key to develop functional as well as effective haptic hardware. The human centered design paradigm leads in more effective stimulation of haptic hardware that mechanoreceptors to visualize the required sensations. The primary focus of this article to extract the research questions from the study and to answer those questions from the driven results and how important is each factor of graspable, wearable and touchable.

In order to identify the evaluation known factors in order to evaluate the performance factors in haptic devices. Many research papers have discussed various methodologies to evaluate them. In early period of haptic touch, the touchable devices were the most dominating technology with passage of time the dominated factor changed to the graspable devices they quite popular in gaming joysticks. In recent times the wearable devices have been introduced and using in haptics [50]. The Virtual Reality is dominated by the wearable devices. based on the study it has been found that the usage of wearable haptic

devices is increased from 2015 and onwards and the rest two categories still have impacted the touchable devices are being used widely for sense of touch purposes [19]. The graspable devices have a smaller number of users and impact over the two categories.

#### 4. Conclusion

The role of the technology has seen such a dominating aspect in communication, online shopping and entertainment increases and growing number of potential applications of sense of touch. The haptics has gained the promising familiarity in terms of usage in mobile communication, gaming as well as in real world applications, although the limit in commercial haptic devices is seen. The impact of the haptics has increased especially in those areas where physical sense of touch is critical to perform the task (e.g., remote surgeries), where touch can be replaced and suggests in terms of visual or auditory cues (e.g., mobile communication and navigation), and where touch can enhance virtual interactions (e.g., virtual reality and gaming). It has been observed that the wearable devices are used most preferred in above cases and the reason is that it is very easy to wear and its hardware can easily be found in the market.

Although limitations of using haptic technology includes lack of availability of hardware but still, haptic, “sense of touch” technology, is expected to gain prominence in the near future as it facilitates human-computer interaction and offers better contact and detailed information about a real-world object. The prolific use of this technology in automotive, education, and gaming is likely to create new opportunities for the global technology market. In addition, the demand for innovative consumer electronics products such as smartphones and tablets are also expected to be a key driver of market growth.

#### REFERENCES

- [1] Chu V, McMahon I, Riano L, McDonald CG, He Q, et al. 2015. Robotic learning of haptic

adjectives through physical interaction. *Robot. Auton. Syst.* 63:279–92

- [2] Flesher SN, Collinger JL, Foldes ST, Weiss JM, Downey JE, et al. 2016. Intracortical microstimulation of human somatosensory cortex. *Sci. Transl. Med.* 8:361ra141
- [3] Johansson RS, Flanagan JR. 2009. Coding and use of tactile signals from the fingertips in object manipulation tasks. *Nat. Rev. Neurosci.* 10:345–59
- [4] Johnson KO, Yoshioka T, Vega-Bermudez F. 2000. Tactile functions of mechanoreceptive afferents innervating the hand. *J. Clin. Neurophysiol.* 17:539–58
- [5] Bolanowski SJ, Gescheider GA, Verrillo RT. 1994. Hairy skin: psychophysical channels and their physiological substrates. *Somatosens. Motor Res.* 11:279–90
- [6] Yohanan, Steve, and Karon E. MacLean. "The role of affective touch in human-robot interaction: Human intent and expectations in touching the haptic creature." *International Journal of Social Robotics* 4.2 (2012): 163-180
- [7] Bushra, N., Carruth, D., & Deb, S. (2018). A Comparative Study of Virtual UI for Risk Assessment and Evaluation. In *Advances in Visual Computing* (pp. 226–236). Springer International Publishing.
- [8] Goedschalk, L., Bosse, T., & Otte, M. (2018). Get Your Virtual Hands Off Me! – Developing Threatening IVAs Using Haptic Feedback. In *Communications in Computer and Information Science* (pp. 61–75). Springer International Publishing.
- [9] Zhao, Y., Ham, J., & van der Vlist, J. (2018). Persuasive Virtual Touch: The Effect of Artificial Social Touch on Shopping Behavior in Virtual Reality. In *Symbiotic Interaction* (pp. 98–109). Springer International Publishing
- [10] Barreiros, J., Claire, H., Peele, B., Shapira, O., Spjut, J., Luebke, D., ... Shepherd, R. (2019). Fluidic Elastomer Actuators for Haptic Interactions in Virtual Reality. *IEEE Robotics and Automation Letters*, 4(2), 277–284.
- [11] Chaudhuri, S., & Priyadarshini, K. (2017). Cultural Heritage Objects: Bringing Them Alive Through Virtual Touch. In *Digital*

- Hampi: Preserving Indian Cultural Heritage (pp. 337–354). Springer Singapore.
- [12] Culbertson, H., Nunez, C. M., Israr, A., Lau, F., Abnoui, F., & Okamura, A. M. (2018). A social haptic device to create continuous lateral motion using sequential normal indentation. In 2018 IEEE Haptics Symposium (HAPTICS). IEEE.
- [13] Haritaipan, L., Hayashi, M., & Mougnot, C. (2018). Design of a Massage-Inspired Haptic Device for Interpersonal Connection in Long-Distance Communication. *Advances in Human-Computer Interaction*, 2018, 1–11.
- [14] Nakanishi, J., Sumioka, H., & Ishiguro, H. (2018). Virtual Hug Induces Modulated Impression on Hearsay Information. In *Proceedings of the 6th International Conference on Human-Agent Interaction - HAI '18*. ACM Press.
- [15] Dombrowski, M., & Dombrowski, J. (2017). Virtual Reality Games, Therapeutic Play and Digital Healing. In *Distributed, Ambient and Pervasive Interactions* (pp. 587–596). Springer International Publishing
- [16] Lee, Y., & Kim, G. J. (2017). Vitty: Virtual Touch Typing Interface with Added Finger Buttons. In *Lecture Notes in Computer Science* (pp. 111–119). Springer International Publishing.
- [17] Kim, H., Kim, M., & Lee, W. (2016). HapThimble. In *Proceedings of the 2016 CHI Conference on Human Factors in Computing Systems - CHI '16*. ACM Press.
- [18] Hoshina, A., Okada, Y., Giannopulu, I., & Sugaya, M. (2016). Haptic Virtual Approach: Biological Effect on Touching and Viewing. In *Lecture Notes in Computer Science* (pp. 261–270). Springer International Publishing
- [19] Cheok, A. D., & Pradana, G. A. (2015). Virtual Touch. In *Scholarpedia of Touch* (pp. 837–849). Atlantis Press.
- [20] Yarosh, S., Mejia, K., Unver, B., Wang, X., Yao, Y., Campbell, A., & Holschuh, B. (2017). SqueezeBands. *Proceedings of the ACM on Human-Computer Interaction*, 1(CSCW), 1–18.
- [21] Takahashi, H., Ban, M., Osawa, H., Nakanishi, J., Sumioka, H., & Ishiguro, H. (2017). Huggable Communication Medium Maintains Level of Trust during Conversation Game. *Frontiers in Psychology*, 8
- [22] Shiomi, M., Nakata, A., Kanbara, M., & Hagita, N. (2017). A hug from a robot encourages prosocial behavior. In 2017 26th IEEE International Symposium on Robot and Human Interactive Communication (RO-MAN). IEEE.
- [23] Mejia, K., & Yarosh, S. (2017). A Nine-Item Questionnaire for Measuring the Social Disfiance of Mediated Social Touch Technologies. *Proceedings of the ACM on Human-Computer Interaction*, 1(CSCW), 1–17.
- [24] Marshall, J., & Tennent, P. (2017). Touchomatic. In *Proceedings of the 2017 Conference on Designing Interactive Systems - DIS '17*. ACM Press
- [25] Sajadieh, S. (2017). Touch Me Here. In *Proceedings of the 2017 ACM on Multimedia Conference - MM '17*. ACM Press.
- [26] Gugenheimer, J., Dobbstein, D., Winkler, C., Haas, G., & Rukzio, E. (2016). FaceTouch. In *Proceedings of the 29th Annual Symposium on User Interface Software and Technology - UIST '16*. ACM Press
- [27] Ahmed, I., Harjunen, V., Jacucci, G., Hoggan, E., Ravaja, N., & Spapé, M. M. (2016). Reach out and touch me: effects of four distinct haptic technologies on affective touch in virtual reality. In *Proceedings of the 18th ACM International Conference on Multimodal Interaction - ICMI 2016*. ACM Press.
- [28] Ahmed, I., Harjunen, V., Jacucci, G., Hoggan, E., Ravaja, N., & Spapé, M. M. (2016). Reach out and touch me: effects of four distinct haptic technologies on affective touch in virtual reality. In *Proceedings of the 18th ACM International Conference on Multimodal Interaction - ICMI 2016*. ACM Press.
- [29] Rasool, S., Hou, X., Liu, Y., Sourin, A., & Sourina, O. (2016). Assessing haptic video interaction with neurocognitive tools. In 2016 IEEE International Conference on Systems, Man, and Cybernetics (SMC). IEEE.
- [30] Misawa, K., & Rekimoto, J. (2016). Who am I Touching? In *Proceedings of the Fourth International Conference on Human Agent Interaction - HAI '16*. ACM Press
- [31] Canat, M., Tezcan, M. O., Yurdakul, C., Tiza, E., Sefercik, B. C., Bostan, I., ... Özcan, O. (2016). Sensation. In *Proceedings of the 2016 CHI Conference on Human Factors in Computing Systems - CHI '16*. ACM Press.

- [32] Maike, V. R. M. L., de Sousa Britto Neto, L., Baranauskas, M. C. C., & Goldenstein, S. K. (2014). Seeing through the Kinect: A Survey on Heuristics for Building Natural User Interfaces Environments. In *Universal Access in Human-Computer Interaction. Design and Development Methods for Universal Access* (pp. 407–418). Springer International Publishing.
- [33] Huisman, G., Bruijnes, M., Kolkmeier, J., Jung, M., Darriba Frederiks, A., & Rybarczyk, Y. (2014). Touching Virtual Agents: Embodiment and Mind. In *Innovative and Creative Developments in Multimodal Interaction Systems* (pp. 114–138). Springer Berlin Heidelberg.
- [34] Wang, J., Vasisht, D., & Katabi, D. (2014). RF-IDraw. *ACM SIGCOMM Computer Communication Review*, 44(4), 235–246.
- [35] Mateu, J., Lasala Bello, M. J., & Alamán, X. (2014). Virtual Touch Book: A Mixed-Reality Book for Inclusive Education. In *Ubiquitous Computing and Ambient Intelligence. Personalisation and User Adapted Services* (pp. 124–127). Springer International Publishing.
- [36] Mei, K., Feng, S., Li, G., Chou, W., & Zhang, L. (2013). A virtual touch event method using scene recognition for digital television. *IEEE Transactions on Consumer Electronics*, 59(1), 214–219.
- [37] Corbett, B., Yamaguchi, T., Liu, S., Huang, L., Bahn, S., & Nam, C. S. (2013). Influence of Haptic Feedback on a Pointing Task in a Haptically Enhanced 3D Virtual Environment. In *Human-Computer Interaction. Interaction Modalities and Techniques* (pp. 561–567). Springer Berlin Heidelberg.
- [38] Monnai, Y., Hasegawa, K., Fujiwara, M., Yoshino, K., Inoue, S., & Shinoda, H. (2014). HaptoMime. In *Proceedings of the 27th annual ACM symposium on User interface software and technology - UIST '14*. ACM Press.
- [39] Angelini, L., Khaled, O. A., Caon, M., Mugellini, E., & Lalanne, D. (2014). Hugginess. In *Proceedings of the 2014 ACM International Symposium on Wearable Computers Adjunct Program - ISWC '14 Adjunct*. ACM Press.
- [40] Overmars, S., & Poels, K. (2013). Virtual Touch Sensations in an Online Shopping Context: An Experimental Approach. In *2013 Humaine Association Conference on Affective Computing and Intelligent Interaction*. IEEE.
- [41] Huang, Y.-P., Wang, G.-Z., Chang, T.-S., & Chen, T.-H. (2013). Three-Dimensional Virtual Touch Display System for Multi-User Applications. *Journal of Display Technology*, 9(11), 921–928.
- [42] Philpott, M., & Summers, I. R. (2012). Evaluating a Multipoint Tactile Renderer for Virtual Textured Surfaces. In *Haptics: Perception, Devices, Mobility, and Communication* (pp. 121–126). Springer Berlin Heidelberg.
- [43] Cuartielles, D., Göransson, A., Olsson, T., & Stenslie, S. (2012). Mobile Haptic Technology Development through Artistic Exploration. In *Haptic and Audio Interaction Design* (pp. 31–40). Springer Berlin Heidelberg.
- [44] Cheok, A. D. (2010). Huggy Pajama: A Remote Interactive Touch and Hugging System. In *Art and Technology of Entertainment Computing and Communication* (pp. 161–194). Springer London.
- [45] Bickmore, T. W., Fernando, R., Ring, L., & Schulman, D. (2010). Empathic Touch by Relational Agents. *IEEE Transactions on Affective Computing*, 1(1), 60–71.
- [46] Zhao, Y., & Dai, S. (2009). A Robust and Fast Monocular-Vision-Based Hand Tracking Method for Virtual Touch Screen. In *2009 2nd International Congress on Image and Signal Processing*. IEEE.
- [47] Ahn, Y.-K., Park, Y.-C., Choi, K.-S., Park, W.-C., Seo, H.-M., & Jung, K.-M. (2009). TOF Depth Camera Based 3D Gesture Interaction System. In *Advances in Information Technology* (pp. 11–17). Springer Berlin Heidelberg.
- [48] Bickmore, T. W., Fernando, R., Ring, L., & Schulman, D. (2010). Empathic Touch by Relational Agents. *IEEE Transactions on Affective Computing*, 1(1), 60–71.
- [49] Bailenson, J. N., & Yee, N. (2007). Virtual interpersonal touch: Haptic interaction and copresence in collaborative virtual environments. *Multimedia Tools and Applications*, 37(1), 5–14.
- [50] Lee, S. P., Cheok, A. D., James, T. K. S., Debra, G. P. L., Jie, C. W., Chuang, W., &

- Farbiz, F. (2005). A mobile pet wearable computer and mixed reality system for human-poultry interaction through the internet. *Personal and Ubiquitous Computing*, 10(5), 301–317
- [51] Bailenson, J. N., & Yee, N. (2007). Virtual interpersonal touch: Haptic interaction and copresence in collaborative virtual environments. *Multimedia Tools and Applications*, 37(1), 5–14.
- [52] Konyo, M., Tadokoro, S., Yoshida, A., & Saiwaki, N. (2005). A tactile synthesis method using multiple frequency vibrations for representing virtual touch. In 2005 IEEE/RSJ International Conference on Intelligent Robots and Systems. IEEE.
- [53] Koskela, T., & Vilpola, I. (2004). Usability of MobiVR Concept: Towards Large Virtual Touch Screen for Mobile Devices. In *Mobile Human-Computer Interaction - MobileHCI 2004* (pp. 252–263). Springer Berlin Heidelberg.
- [54] Dionisio, J., Henrich, V., Jakob, U., Rettig, A., & Ziegler, R. (1997). The virtual touch: Haptic interfaces in virtual environments. *Computers & Graphics*, 21(4), 459–468.
- [55] Sreelakshmi, M., and T. D. Subash. "Haptic technology: A comprehensive review on its applications and future prospects." *Materials Today: Proceedings* 4.2 (2017): 4182-4187.
- [56] El Saddik, Abdulmotaleb. "The potential of haptics technologies." *IEEE Instrumentation & Measurement Magazine* 10.1 (2007): 10-17.
- [57] D'Auria, Daniela, Fabio Persia, and Bruno Siciliano. "A low-cost haptic system for wrist rehabilitation." 2015 IEEE International Conference on Information Reuse and Integration. IEEE, 2015.
- [58] Liu, Lu-miao, Wei Li, and Jia-jia Dai. "Haptic technology and its application in education and learning." 2017 10th International Conference on Ubi-media Computing and Workshops (Ubi-Media). IEEE, 2017.
- [59] Agboh, Wisdom C., Mustafa Yalcin, and Volkan Patoglu. "A six degrees of freedom haptic interface for laparoscopic training." 2016 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS). IEEE, 2016.
- [60] Muis, Abdul, and Kouhei Ohnishi. "Cooperative mobile manipulator with dual compliance controllers based on estimated torque and visual force." 2006 IEEE International Conference on Mechatronics. IEEE, 2006.
- [61] Salisbury, Kenneth, Francois Conti, and Federico Barbagli. "Haptic rendering: introductory concepts." *IEEE computer graphics and applications* 24.2 (2004): 24-32.

0.7" ↑ Margin Top

## Paper Formatting Guidelines

**Title: 14<sup>pts</sup> Bold**

**Author Name: 11<sup>pts</sup> Bold**

*Affiliation: 11<sup>pts</sup> Italic*

**Abstract:** Single Paragraph (Min: 150 – Max: 250 words)

**Paper Length:** Formatted as guided (Min: 4,000 – Max: 8,000 words)

**Font:** Times New Roman 10<sup>pts</sup>

**Font Color:** Black

**Line Spacing:** 1.0 (single line space throughout the paper)

**Indentation:** Justify

**References & In-text Citations:** Follow the IEEE & Use EndNote X7 for In-text citations and Bibliography.

**Headings: 12<sup>pts</sup> Bold**

**1. 12<sup>pts</sup> Bold**

**1.1. 11<sup>pts</sup> Bold**

**1.1.1. 11<sup>pts</sup> Bold Italic**

← Margin Left  
0.8" →

← Margin Right  
0.5" →

**6.5" x 10"**

*Page Size: 6.5" (Width) x 10" (Height)*

**Submission:** Formatted paper as guided can be submitted through our online submission system at <http://sjcms.iba-suk.edu.pk>

1.9" ↑ Margin Bottom  
↓

# Sukkur IBA **Journal** of Computing and Mathematical Sciences



**SUKKUR IBA UNIVERSITY**  
Merit - Quality - Excellence

**SUKKUR IBA UNIVERSITY**  
Airport Road, Sukkur -65200  
Sindh, Pakistan  
Tel: +92-71-5644233  
Fax: +9271-5804419  
Email: [sjcms@iba-suk.edu.pk](mailto:sjcms@iba-suk.edu.pk)  
URL: [sjcms.iba-suk.edu.pk](http://sjcms.iba-suk.edu.pk)

## HEC Recognized Journal